DECIMAL ARITHMETIC MADE PERFECT;

OR, THE

Management of Infinite Decimals DISPLAYED.

Being the Whole Doctrine of the Arithmetic of Circulating Numbers, explained by many New and Curious Examples in ADDITION, SUBSTRACTION, &c. Of all which the last Age was entirely ignorant, but now made Easy and Familiar to the meanest Capacity. With proper Demonstrations to illustrate the Whole; in a Manner hitherto Unattempted, or at least not Published by any Author.

To which is Prefixed,

An HISTORICAL INTRODUCTION, shewing the Progress and Improvements made therein by its several Authors, from the very First Attempt down to the Present Time.

With LARGE TABLES annexed to compleat the Whole.

AND AN

APPENDIX,

CONTAINING

The ARITHMETIC of the Five Primary RULES in Decimal Fractions, as commonly Taught.

By *JOHN MARSH*, Writing-Master, and Accomptant, in the City of *Sarum*.

K

LONDON:

Printed for the AUTHOR; and Sold by EDWARD EASTON, and BENJAMIN COLLINS, Booksellers in *Sarum*; and JOHN and PAUL KNAPTON, in *Ludgate-Street*, *London*. 1742.

TO THE

RIGHT HONOURABLE

HENRY

EARL of

PEMBROKE and *MONTGOMERY*,

One of His MAJESTY'S most Honourable Privy-Council, &c.

AND

Lord High Steward of the City of NEW-SARUM.

As a great Encourager of ARTS and SCIENCES,

This TREATISE,

OF

DECIMAL ARITHMETIC made Perfect,

Is, with all Humility, most humbly Presented and Dedicated,

BY,

My LORD,

Your Lordship's

most obedient Servant,

J. MARSH.

THE

PREFACE.

I HAVE in the Introduction and in the following Book ſaid ſo much concerning Infinite Decimals, and the Management of them in Arithmetical Calculations, that I think I may be very juſtly excuſed from making a long Preface.. And indeed were it not to have comply'd more with Cuſtom, than with my own Inclination, I ſhould have made none at all. However, to gratify ſome Readers by the Way of Prefacing, I choſe here to place the following Remarks.

1. The univerſal Application of Decimal before all other Kind of Fractions to every Branch of Mixt Mathematics, beſpeaks their Superlative Excellency, more than all the labour'd Periods of Encomium can poſſibly do. Wherefore I am inclin'd to believe, that I ſhall be readily excuſed, if I ſpare myſelf the Pains of making any on their general Uſefulneſs, or indeed of giving myſelf the Trouble (by long Harangues) of recommending to my Reader the abſolute Neceſſity, that every Arithmetician lies under, of perfectly knowing the moſt exact Way of managing them.

2. But however, I ſhall take the Liberty in this Place to make the following Obſervation; *viz.* That no Perſon, who is ignoraut of the Arithmetic of Infi-

nite Decimals, can be ſaid to underſtand Decimal Arithmetic perfectly well; becauſe without its Aſſiſtance the Reſult of his Operations muſt generally be imperfect, and the Error very conſiderable too, when he deals with large Numbers. For Inſtance;

Let us ſuppoſe the following Finite Mixt Number, *viz.* 96,75, was given to be multiplied by ,0$\dot{6}$, where 6 would infinitely repeat from the Place of Hundredths of an Unit.

Now in Conſideration that the Integral Number in the Multiplicand is ſo little as 96, and the Multiplier in appearance is ſo diminitive as ,06, from thence I readily believe that almoſt every Practitioner in Common Decimals would be content to give their Product as with two Finite Expreſſions, which is 5,805; whereas its Mathematical exact Product is 6,45 : So that the Defect of the Former would be ,645, which is too little by juſt the one Ninth of its Common Product. And if ſo conſiderable an Error will ariſe from ſuch ſmall Numbers, as above, how great may the Defect be, when we deal with very large Numbers! For, unleſs the Practitioner in common Decimals be careful to make every Approximate Factor to conſiſt of Eight or more Figures deep in its Fractional Part, the Error will be very conſiderable. And indeed let him, if he pleaſe, make them Millions of Figures deep, yet after all his Labour the Reſult will be imperfect. Whereas the following Sheets will inſtruct him how to find the Reſult mathematically exact in a very narrow Compaſs.

3. Who therefore, among ſuch as deſire to be eſteemed Compleat Arithmeticians, would now continue longer ignorant of the Arithmetic of Infinite Decimals? Which the following Tract, I do not doubt, will render very eaſy and familiar even to every common Capacity: And that too upon the Principles of Vulgar Fractions only, without having recourſe to any Complext Algebraical Theorem for its Aſſiſtance.

4. It being natural for the Reader to expect, according to the general Cuſtom of Prefaces, that the Author ſhould ſomewhere in this Place give him a ſuccinct Account of the Particulars which he may meet with in the Body of his Work, ſo, whoever will turn to the Contents of this Book, he will there find a very ample Account of the Order or Succeſſion of the ſeveral Parts of the whole Compoſition; to which therefore, to avoid Prolixity, I muſt here refer my Reader.

5. As for my Stile, I have endeavoured to make it as plain and uniform, as the Nature of the Subject will give leave. And for the Redundancies (or if my Readers rather chuſe to call them Tautologies) which are here and there to be met with, I hope they will turn out an Advantage to the mere *Engliſh* Scholar. For I have for many Years experienced, that where different Rules have been delivered in the leaſt Variety of Diction, there Youth in general have made the quickeſt Progreſs.

Wherefore, as I deſign'd my Compoſition for the Uſe of the moſt Illiterate, I have been the more careful to uſe as few Variations in Expreſſion as agreeably with plain common Senſe I well could.

6. And

6. And laſtly, I hope that throughout the Whole there are no material Faults. If upon Peruſal any ſuch ſhould appear, I ſhall be very thankful to that Perſon who ſhall be ſo kind as to apprize me of them. Such ſmall Errata as commonly attend both Pen and Preſs, in Numerical Books eſpecially, I doubt not but that every unprejudiced Reader will candidly excuſe and correct.

To avoid any Miſconſtruction, the Reader, before he peruſes the Book, is deſired to correct the following Errata.

IN the Introduction, Page vii. line 21. *for* Gentleman *read* Gentlemen. In the Book, P. 14. l. 16, 17. *for* =2754753 *r.* =2754726. P. 22. l. 14. *for* Expreſſions *r.* Expreſſion. P. 39. l. 3. *for* $\frac{3899}{1056}$ *r.* $\frac{389}{1056}$. P. 52. l. 21. remove the Speck from 5 to its next Figure 9. P. 53. l. the laſt, *for* 12345678 *r.* 123456788. P. 59. l. 12. place a Speck over the firſt Place in Decimals. P. 61. l. 7. *r. Ex.* 3. Is. P. 65. *Ex.* 4. *for* ,00397317 *r.* ,00307317. P. 70. l. 2. *r.* multiply ,6 by ,8. P. 77. l. 15. the Number 112500 ſhould have been ſet one Place more towards the Left-hand. P. 80. l. 20. *for* 32,00 *r.* 320,0. P. 99. l. 20. *for* 22 *r.* 22. P. 110. l. 8. dele the Speck over the latter 7. P. 112. l. 14. *for* 78,048 *r.* 78,048. line the laſt, *for* 878,04 *r.* 878,04. P. 118. l. 4. *for* 57945 *r.* 57945. P. 150. l. 3. *for* ,00411 *r.* ,00411. P. 151. l. 7. *for* ſquared *r.* Square. P. 154. l. 16. *for* ,45 *r.* ,45. P. 156. l. 13. *for* ,3 *r* ,3. P. 159. l. 3. *for* ,81 *r.* ,81. P. 160. l. 12. *for* ,8846153 *r.* ,8846153. P. 162. l. 8. *for* ,1714285 *r.* 1714285.

THE

THE CONTENTS.

THE INTRODUCTION, Page

CHAP. I.

Definitions and Propositions, 1

1. *What a Repetend or Circulate is,* ibid.
2. *Repetends are either Single or Compound,* ibid.
3. *What a Single Repetend is,* ibid.
4. *What a Compound one,* 2
5. *Repetends in general are either Pure or Mixt,* ibid.
6. *What a Pure Repetend is,* ibid.
7. *What a Mixt one,* ibid.
8. *What the Finite and Circulating Parts are of any Mixt Repetend,* 3
9. *How to find a Vulgar Fraction Equivalent to any given Repetend,* 5
10. *How to multiply any given Number by any Number of 9's most compendiously,* 10
11. *How to do the same when any given Number is to be added to its Product,* 11
12. *The shortest Method how to find the Equivalent Vulgar Fraction of any Mixt Repetend,* 13
13. *Shews the Value of a Repetend when* $,\dot{9}$ *or* $\dot{9}$, &c. 16

14, 15, 16. *Shew when the Denominators of any Vulgar Fractions are 9 or 9's with or without 0's, what their Equivalent Repetends are,* 16, 17

 17. *The*

CONTENTS.

17. *The Method of transforming a given Repetend to another Equivalent one, having the same or a greater Number of Places of Figures, with their several Demonstrations,* 18
18. *Repetends are either Similar or Dissimilar,* 24
19. *What a Similar one is,* ibid.
20. *What a Dissimilar one is,* ibid.
21. *How to transform two or more Dissimilar Repetends to Similar ones,* ibid.
22. *What the least Common Multiple to two or more Numbers is,* 25
23. *How to find the least Common Multiple to two or more given Numbers,* ibid.
24. *How to make a Finite or Determinate Number become a Similar Circulating Expression.* 29

CHAP. II.

Addition of Circulates, 30

CHAP. III.

Subtraction, 41
25. *How to multiply any Single or Compound Repetend by* 10, *or* 100, *or* 1000, &c. 48
26. *The most Compendious Method of dividing any Number by any proposed Number of* 9's, 50

CHAP. IV.

Multiplication, 56

CHAP. V.

Division, 90
27. *The most Compendious Way of multiplying any Repetend by any Number of* 9's, 105

CHAP.

CONTENTS.

CHAP. VI.

Reduction, 120

CHAP. VII.

Involution and Evolution, 132

28. *What Involution is*, ibid.
29. *Its Rule*, ibid.
30. *What Evolution is*, 145
31. *All Powers above the first are either Rational or Irrational*, ibid.
32. *What Rational Powers are*, ibid.
33. *What Irrational Powers are*, ibid.

And lastly, The Tables with their Explanation and Use, 154 &c.

An EXPLANATION *of the* CHARACTERS *and* ABBREVIATIONS *made use of in the following Sheets.*

CHARACTERS.	NAMES.	SIGNIFICATIONS.
$=$	Equal to.	The Mark of Equality. As $\overset{L.}{1} = \overset{S.}{20}$.
$+$	Plus, or More.	The Mark of Addition. As $9 + 6 = 15$; read 9 plus or more 6 are equal to 15.
$-$	Minus, or Less.	The Mark of Subtraction. As $9-6=3$; read 9 minus or less 6 is equal to 3.
$\times$	Multiplied by.	The Mark of Multiplication. As $9\times 6 = 54$; read 9 multiplied by or into 6, is equal to 54.
$\div$	Divided by	The Mark of Division. As $9 \div 6$ (or thus $\frac{9}{6}$) $= 1\frac{1}{2}$; read 9 divided by 6 is equal to $1\frac{1}{2}$.
$\surd$	Square Root.	As $\surd\, 36 = 6$; read the Square Root of 36 is equal to 6.

Hence then $\frac{2\times 9 + 7}{90} = \frac{25}{90}$ is read thus; 2 multiplied by 9, to whose Product add 7, and that Sum divided by 90, is equal to 25 divided by 90.

ABBRE-

ABBREVIATIONS.

Num$^{r.}$ *for* Numerator.

Denom$^{r.}$ *for* Denominator.

E. S. F. *for* Equivalent Single Fraction or Fractions.

E. V. F. *for* Equivalent Vulgar Fraction.

C. P. *for* Common or First Product.

In making uſe of the above Symbols or Characters, we avoid many and frequent Repetitions of the ſame Words. And by it alſo we have this farther Advantage, *viz.* of compriſing the whole Subject in, or nearly with, two Thirds of the Paper that the verbal Way would neceſſarily require.

A CA-

A
L I S T
OF THE
NAMES of as many of the SUBSCRIBERS to
DECIMAL ARITHMETIC made Perfect,
As came in Time to be inserted.

A

AMbrose Awdry *of* Seend, *Gent.*
Mr. William Andrews *of* Ambrosbury, *Writing-Master.*
Mr. Samuel Allen *of the* Dock, Portsmouth.
The Reverend Mr. Dummer Andrews.
The Rev. Mr. Abbot, *A. M. Fellow of Baliol College*, Oxon.

B

Mr. James Brown, *Gardener to the Right Honourable the Earl of* Pembroke.
Mr. Samuel Biggs *of the* Park.
Henry Biggs *of* Woodford, *Gent.*
Thomas Bennet *of* Salthrop, *Esq*;
James Bartlett *of* Sarum, *Gent.*
Mr. P. M. Broheir *of* Southton.
Thomas Baskerville *of* Lower Richardson, *Esq*;
Thomas Bowles, *D. D. Rector of* Brackley *in* Northamptonshire.
The Reverend Mr. Balguy, *M. A. Fellow of St. John's College* Cambridge.
Mr. William Bryant *of St. John's College* Cambridge.
The Reverend Mr. Barford, *A. B. Fellow of King's College* Cambridge.
Mr. John Banson *of* Trinity Hall, Cambridge.
John Barker, *M. B. of the Close of* Sarum.
John Thomas Batt, *M. B. of Baliol College*, Oxon; *and F. R. S.*

Mr.

Mr. Thomas Bucknall, *Assistant Builder in his Majesty's Dock*, Portsmouth.
Arthur Batt, *Esq*; *of Baliol College*, Oxon.
The Reverend Mr. Blake *of* Sarum.
Mr. Thomas Bacon *of* Sarum.
The Reverend Mr. George Bowditch.
The Reverend Mr. William Bowles, *A. B. Fellow of New College*, Oxon.
The Reverend Mr. Bradley, *Professor of Astronomy in University College*, Oxon.
The Rev. Mr. Bliss, *Professor of Geometry in University College*, Oxon.
Mr. Brickland, *Writing-Master in* Oxon.
Mr. Bayles *of Pembroke College*, Oxon.
Brydges Blackford *of* Osborne *in the Isle of* Wight, *Esq*;
Master James Benjafield *of* Blandford.
Mr. William Best *of* Whitechurch, Dorset.
Mr. Henry Brown *of* Sarum, *Apothecary*.
John Ballard, *D. D. Rector of* Langford Magna.
Thomas Bartlett *of* Cranbourne, *Gent*.
Mr. Charles Baker, *School-Master in* Wareham.

C

The Reverend Mr. Joseph Cuthbert.
Mr. Richard Corpe *of* Poole, *Writing-Master*.
The Reverend Thomas Yale Caverley, *Rector of* Foley, Hants, *and Chaplain to his Royal Highness the Prince of* Wales.
The Reverend Thomas Clifton, *D. D. Rector of* Boyton.
The Reverend Mr. Colson, *Professor of Mathematics in* Cambridge.
Francis Cust, *Esq*; *Fellow of King's College*, Cambridge.
John Coker, *L. B. of New College*, Oxon.
The Reverend Henry Coker, *L.B. Fellow of New College*, Ox.
The Rev. Thomas Coker, *A. M. Fellow of New College*, Ox.
The Reverend Mr. Camplin, *Vice Principal of Edmund Hall*, Oxon.
Mr. Combe *of Merton College*, Oxon.
The Reverend Mr. Coates *of Magdalen Hall*, Oxon.
Mr. John Cuel.
Mr. Thomas Chubb *of* Winchester, *Accomptant*.
Mr. William Clarke.

Mr.

Mr. Robert Cooper, *Junr. of* Sarum.
Mr. Benjamin Collins, *Bookſeller in* Sarum. Seven Books.
Mr. John Collis, *Carpenter in* Redbridge.

D

The Reverend Mr. Rowland Dennis, *A. M. of* Sarum.
Charles Delafaye, *Eſq; of* Wichbury.
Henry Deacon, *Eſq;*
Mr. Davies, *Writing-Maſter in* Warminſter.
Mr. Thomas Roger du Queſne, *Fellow of King's College,* Cambridge.
Lewis Dechair *of the Inner Temple, Eſq;*
Mr. Richard Dignell, *Gardener to the Right Honourable Lord* Carberry *at* Woolſton.
Mr. James Darke *of* Honiton, *Writing-Maſter.*
Mr. Walter Dench *of* London.
The Rev. Mr. Drake, *A. M. Fellow of Baliol College,* Oxon.

E

Scoope Egerton, *Eſq;*
The Reverend Mr. Eyre, *Junr. Fellow of New College,* Ox.
Mr. Edward Eaſton, *Bookſeller in* Sarum. Two Books.
The Rev. Samuel Eyre, *LL. D. Rector of* Broughton, Hants.

F

John Fullerton, *Eſq;*
The Reverend Mr. Foſter, *M. A. of the* Cloſe *of* Sarum.
The Reverend Mr. Fletcher, *A. B. Fellow of New College,* Oxon.
Mr. Froſt *of Exeter College,* Oxon.
Mr. John Fletcher, *Bookſeller in* Oxford. Two Books.
Dr. Frampton, *M. D. of* Oxford.

G

Matthew Goodfellow *of* Dinton, *Gent.*
Mr. Abraham Gapper.
The Reverend Mr. Green, *M. A. Fellow of St. John's College,* Cambridge.
The Reverend Mr. Golding, *LL. B. Fellow of New College,* Oxon.
The Rev. Mr. Godwyn, *D. B. Fellow of Baliol Coll.* Oxon.
Mr. Thomas Glover *of the* Dock, Portſmouth.

H

James Harris *of the* Cloſe *of* Sarum, *Eſq;*
Thomas Harris *of* Lincoln's-Inn, *Eſq;*

The Reverend Mr. George Harris, *Prebendary of* Sarum.
William Harris *of the* Close *of* Sarum, *Esq*;
The Rev. Mr. Harris, *A. M. Fellow of New College*, Oxon.
Mr. William Hulbert, *Mercer in* Sarum.
Mr. John Hockley *of* Southton.
Mr. William Huse *of* Shaston.
William Hiley *of* Upton *near* Poole, *Esq*;
Mr. Hackett.
William Heberden, *M. D. Fellow of St. John's Coll.* Camb.
The Reverend Mr. Hill, *M. A. of St. John's College*, Camb.
Mr. Huddlestone, *Teacher of the Mathematics in* Coventry.
William Hillman, *M. B. of Baliol College*, Oxon.
Mr. Thomas Hayter, *Mercer in* Sarum.
The Rev. Mr. Hoadley, *Chancellor of the Diocese of* Winchester.
William Hunt, *Esq*; *of* West Lavington.
Mr. J. Hayne *of* Axminster, *Writing-Master.*
The Reverend Mr. Richard Hele *of* Chardstocke.
The Reverend Mr. Joshua Fisher Harrison.
Mr. William Hussey *of* Sarum.
Sir Thomas Hoby, *Bart.*
Mr. John Hicks, *Attorney at Law.*

I

Mr. Jope, *M. A. Fellow of New College*, Oxon.
Mr. John Janes, *Attorney at Law.*
William Jones, *Gent.*

K

Morgan Keen *of the* Close *of* Sarum, *Gent.* Two Books.
Francis Kenton *of* Sarum, *Esq*;

L

Captain Southcot Hungerford Lutterell.
Mr. Alderman Light *of* Sarum.
Mr. J. Lloyd *of* Southton, *Writing-Master.*
Robert Lewen *of* Blandford, *Esq*; Five Books.
The Library *of St. John's College*, Cambridge.
The Rev. Mr. Lydiatt, *Senr. A. M. Fellow of New College*, Oxon.
The Rev. Mr. Lumby, *A.M. Rector of St.* Martin *in* Sarum.
Mr. G. Liddell, *Gardener to the Right Honourable Lord* Palmerston *near* Rumsey.

M

Mr. Matthew Merefield *of* Shafton, *Gent.*
Mr. Edward Moody *of* Burford.
Captain John Mackenzie.
The Rev. Mr. Mitchell, *L. B. Fellow of New College*, Oxon.
Mr. William Moore, *Attorney at Law in* Bridg-Water. Six Books.
Mr. Moore *of* Melkfham Wilts.
Mr. Robert Matthews *of* Bath.
Mr. Daniel Martin *of* Southton, *Writing-Mafter.*
Chriftopher Gore Michell, *Efq;*
Cornelius Michell, *Captain of his Majefty's Ship the* Kent.
The Reverend Mr. Thomas Michell.
Mr. Robert Michell.
Mr. Francis Michell *of* London.
Mr. Rober Marfh *of* London.
Mr. William Marfh *of* London.
Mr. Robert Sutton Marfh *of* London.
Mr. Thomas Marfh *of* Briftol.
Mr. Edward Marfh *of* Briftol.

N

The Reverend Dr. Newcombe, *D. D. Mafter of St. John's College*, Cambridge.
Mr. Alderman Naifh *of* Sarum.
The Rev. Mr. Noel, *A. B. Fellow of New College*, Oxon.
The Reverend Mr. John Norris.
Mr. Mitchell Newman.

P

Mr. Green Palmer *of* Sarum.
Mafter Thomas Pope.
The Reverend Mr. David Price *of* Weft Lavington.
Mr. Francis Price *of* Sarum, *Builder.*
The Rev. Mr. Price, *Senr. L.B. Fellow of New Coll.* Oxon.
The Rev. Mr. Price, *Junr. A.M. Fellow of New Coll.* Oxon.
Mr. Haily Pond *of* Shafton, *Writing-Mafter.*
Henry Pincke, *Efq*; *of* Kempfhot, Hants.
Mafter Jonathan Harris Power *of* Sarum.
Mr. William Pitt, *Bookfeller in* Blandford.

Mr.

Mr. Phelps, *Fellow of New College*, Oxon.
Mr. Alexander Powell *of* Sarum, *Attorney at Law.*
Mr. James Praid.
The Rev. Mr. Parker, *A. M. Fellow of Baliol College*, Oxon.

R

Mr. Thomas Reading, *Attorney at Law in* Sarum.
The Reverend Mr. Rutherforth, *M. A. Fellow of St. John's College*, Cambridge.
Mr. John Reepe *of King's College*, Cambridge.
The Rev. Dr. Rothwell, *D. D. Rector of* Monkston.
The Rev. Mr. Reynolds, *Præcentor of* Down *and* Conner *in* Ireland.
The Rev. Mr. Role, *A. M. Fellow of New College*, Oxon.
Mr. James Russell *of* Downton, *Writing-Master.* 2 Books.
Mr. James Russell, *Junr. Writing-Master on the Common*, Portsmouth.
Mr. John Russell *of* Basingstoke, *Attorney at Law.* Two Books,
The Rev. Mr. Russell, *A. M. Fellow of Baliol College*, Oxon.

S

Mr. John Snow *of* Oxford, *Organist.* Two Books.
Francis Swanton *of* Sarum, *Gent.*
The Rev. Mr. Squire, *M. A. Fell. of St. John's Coll.* Camb.
The Rev. Mr. John Squire, *Prebendary and Sub-Chanter of* Sarum.
Mr. John Stewart *of St. John's College*, Cambridge.
Mr. Shelford *of Caius College*, Cambridge.
Mr. J. S. Samber *of St. John's College*, Cambridge.
Mr. Robert Stillingfleet, *Attorney at Law in* Sarum.
Mr. Richard Sydenham.
Dr. Smyth, *M. D. Fellow of New College*, Oxon.
The Rev. Mr. Joseph Sager, *Canon of the Cathedral Church of* Sarum.
Mr. Steward.
Webb Seymour, *Esq*;
William Seymour, *Esq*;
The Reverend Mr. George Shuttleworth.
Mr. Rowland Smith *of the* Dock, Portsmouth.
Mr. Robert Symons *of* Wilton.
Mr. Alderman Stone *of* Sarum.
Mr. Edward Sethcote.

Mr.

T

Mr. George Thornton *of* Grimstead.
Mr. Edward Thompson *of* Sarum, *Organist.*
The Rev. Mr. John Talman, *M. A. of the* Close *of* Sarum.
Alexander Thistlethwayte *of* Winterslowe, *Esq*; Six Books.
Robert Thistlethwayte, *Esq*; of *Wadham College*, Oxon.
Mr. Robert Tarrant, Fordingbridge, *Attorney at Law.* Two Books.
Mr. Richard Taylor *of* Southton.
Mr. Walter Taylor *of* Southton.
Mr. John Tarrant *of* Downton, *Attorney at Law.*

W

John White *of* Shaston, *Esq*;
Mr. Alderman Wentworth *of* Sarum.
James Wroughton *of* Wilcot, *Esq*;
George Wroughton *of* Lincoln's-Inn, *Esq*;
John Wadman *of* Ember, *Esq*;
Mr. George Wooldridge *of* Bishop-Stoke.
The Rev. Dr. Whalley, *Master of Peter-House Coll.* Camb.
The Rev. Mr. Wrigley, *President of St. John's Coll.* Camb.
The Rev. Mr. Wright, *M. A. of St. John's College*, Camb.
Mr. Nicholas Wakeham, *of King's College*, Cambridge.
Richard Wansbrough *of* Trowbridge, *Gent.*
Mr. John Walton, *of the Royal Academy in* Portsmouth.
Mr. John Walsh *of* Thorncomb, Devon.
Mr. Thomas Wallis, *Senr. of* Sarum.
Mr. Joseph Whatley, *Accomptant.*
Mr. Robert Wentworth, *Jeweller and Goldsmith in* Sarum.
Mr. Thomas Wentworth, *Lieutenant of his Majesty's Ship the* Hampshire.
Mr. Thomas Whatley *of* London.
Mr. John Willis *of* East-Orchard, *Writing-Master.* Four Books.
Mr. William Webb, *one of the General Examiners at the Excise Office.*
Mr. William Wapshare, *Land-Surveyor.*

THE

INTRODUCTION.

EVERY Perſon, converſant in Decimal Fractions, muſt have obſerved, that in the turning of a Vulgar Fraction into a Decimal Fraction, it is very rare that the Quotient is finiſhed off without leaving any Remainder.

And they muſt likewiſe have taken notice, that in the Quotient, which turns not out a perfect, determinate, or compleat Decimal Expreſſion, they often find either the ſame Figure, or Figures, continue to occur ſomewhere in the Quotient; and that, if the Diviſion were continued on *ad infinitum*, the ſame Figure, or Figures, would be infinitely repeated in the Quotient. And ſuch Decimal Expreſſions as theſe, are therefore called interminate, indeterminate, or infinite Decimals, in Contradiſtinction to a perfect, determinate, compleat, or finite Decimal Expreſſion.

But as to the Management of infinite Decimals in Arithmetical Operations, by Addition, Subſtraction, &c. the Age (and many Years after) in which Doctor *Wallis* publiſhed his *Hiſtory of Algebra*, which was in the Year 1685, was entirely ignorant of: For the Dr. who wrote the Hiſtory of Decimals, is wholly ſilent therein, not giving ſo much as a Hint at a Method how to add or ſubſtract them, &c. but by Approximation only. Whereas now, by the Improvements lately made, it is poſſible to give the Sum, or Difference, the Product, or Quotient of any Fractions whatſoever, not ſurd Roots, in Decimal Expreſſions mathematically exact.

As I intend this INTRODUCTION as an Hiſtorical Account of the Progreſs, made from Time to Time, in the Management of infinite Decimals, till they lately aroſe to their preſent Perfection, I beg leave here firſt to tranſcribe from the learned Doctor's *Hiſtory of Algebra* part of Chap. 89, which contains his curious Remarks on that of repeating Decimals.

The Dr. having treated, among other mathematical Sciences, of the method of Exhauſtions, and the Arithmetic of Infinites, which depends on that of Exhauſtions, as alſo of the Method of infinite Series, or continual Approximations (grounded on the ſame Principles) ariſing principally from Diviſion and Extraction of Roots in Species infinitely continued; hath in the above Chapter made the following curious Obſervations upon circulating Decimals.

This Diviſion in Species, is much of the ſame Nature (but more univerſal) with that (in Numbers) of reducing common Fractions to Decimals: which ſometimes ends in a determinate Quotient: As $\frac{1}{2}=0,5$: $\frac{1}{5}=0,2$: $\frac{1}{10}=0,1$: $\frac{1}{4}=0,25$: $\frac{1}{8}=0,125$: $\frac{3}{20}=0,15$: $\frac{6}{15}=\frac{2}{5}=0,4$: Which then happens, and only then, when (the Fraction being firſt reduced to the ſmalleſt Terms) the Denominator (or Diviſor) is compounded of no other prime Numbers than 2 and 5, (of which 10 is compounded.)

But if the Denominator (ſo reduced) be compounded of any other prime Number (than 2 or 5) the Quotient will be interminate: As $\frac{1}{3}=0,3333$ &c. $\frac{2}{3}=0,6666$ &c. $\frac{1}{11}=0,0909$ &c. $\frac{2}{11}=0,1818$ &c. $\frac{1}{13}=0,076923076923$ &c. $\frac{4}{27}=0,148148$ &c. $\frac{2}{37}=0,054054$ &c.

In

In which Case there is yet always this Concinnity, that after some time, the Numbers do again return, and circulate in the same Order as before: sometime in a Repetition of one single Figure, (as was seen in $\frac{1}{3}$, $\frac{2}{3}$) Sometime, of two or more, (as in $\frac{1}{11}$, $\frac{2}{11}$, $\frac{1}{13}$, $\frac{4}{27}$, $\frac{2}{37}$) but always, if not sooner, it doth at least begin to return in so many Places as are the Number of Units in the Divisor.

For instance, $\frac{1}{7}$ =0,142857142857 &c. For the Divisor being 7, the Remainder must always be less than it; and therefore 1, 2, 3, 4, 5 or 6: So that in the seventh Place, at least, if not before, one of the Remainders must needs return a second Time: And the same Remainder returning as before, the same Figure or Figures in the Quotient must also return; and so onward.

The Number of Figures therefore which do thus circulate, is never more than the Number of Units in the Divisor, wanting one. But many times, it is only an aliquot Part of such Number, or some lesser Number which is not an aliquot part of it.

And to know when this happens, the Fraction being first reduced to its smallest Terms; and the Denominator of that (reduced) Fraction, being farther reduced, by dividing it by 2 and 5 (the Components of 10) as oft as it can: If then it come to be 9, 99, 999 &c. (consisting only of the Figure 9 repeated,) or an aliquot part of such Number; so many as are the Figures of 9 in such Number which first occurreth, so many are the Figures of such Circulation.

Thus, if the Divisor or Denominator of the Fraction be 9,3,6 (=2×3,) 12 (=2×2×3,) 15(=5×3,) &c. the Circulation is of single Figures; because in 9, that Fi-

 gure

gure is but once written; and 3 is an aliquot Part of 9; and 6,12, 15 are made by Multiplications of 3, by 2, or 5, (the Components of 10.)

If 99,11,22 (=2×11,) 33,55 (=5×11,) 66 (=2×33,) &c. the Circulation is of two Figures, becauſe 99 is denoted by 9 twice written; and 11,33, are aliquot Parts of 99; and 22,55,66, &c. are made by Multiplications of one of them by 2, or 5: (I do not here mention 9, or 3, though theſe alſo be aliquot Parts of 99) becauſe theſe appertain to the former Rank; and therefore admit not only of Circulation by Couples, (but even by ſingle Figures.)

If the Diviſor (ſo reduced) be 999,27,54 (=2×27,) 135 (=5×27,) 37,74, (=2×37) &c. the Circulation for like reaſon is of three Figures. If 13; it is of 6 (the half of 12, which is 13—1,) becauſe 13 doth accurately divide, or is an aliquot part of 999999, wherein 9 is ſix times written; (but not of any Number deſigned by the Figure of 9 fewer times repeated.)

If 21, (which is not a prime Number:) it is of ſix Figures, (which yet is not an aliquot Part of 20=21—1,) becauſe it divides 999999. Or thus, becauſe 21 is a Compound of 3×7; whereof 7 requires (as before) a Circulation of 6 places; but 3 a Circulation only of 1 place, (which is an aliquot Part of 6,) this (ſixtimes repeated) will terminate with one Revolution of 6 places.

And the like of 77=7×11; becauſe 11 requiring but a Circulation of 2 places, (which is alſo an aliquot part of 6,) three of theſe Circulations will terminate with One for the Number 7, which is of 6 places.

So 259=7×37; becauſe 37 requires but a Circulation of 3 places (which is alſo an aliquot Part of 6;) two Circulations of this, will end with One of that for 7. And the like in other Caſes.

But

But if the component prime Numbers, (other than thoſe of 2 and 5, before conſidered,) be ſuch as require Circulations, whereof the one is not an aliquot of the other; then, (though the one be of fewer places, yet) will the compound Circulation be more than that of the ſingle greateſt; namely, of ſo many places as is a Number diviſible by both thoſe for the Components.

As for inſtance; 11 requires a Circulation of 2 places, and 37, one of 3 places; therefore $407=11\times37$, will require one greater than either; namely, one of 6 places, (this being the firſt Number that may be divided by 2 and 3,) that ſo 2 Circulations of the one, may end with 3 of the other.

The like for $297=11\times27$; becauſe 27 (though not a prime Number) requires a Circulation of 3 places.

And the like Eſtimate is to be made for other compounded Numbers.

All which yet is not ſo to be underſtood, as if this Circulation did always take its beginning from the firſt Place of Decimal Fractions. For when the Denominator or Diviſor is compounded of 2, or 5, or any Powers of theſe, it begins not till ſometime after; that is, not till the Influence of thoſe Components ceaſe to operate; that is, not till after ſo many places as is the Number of ſo many Dimenſions of 2 or 5 aſſumed in that Compoſition.

Thus is $\frac{5}{12}=0,41\dot{6}666$ &c. For to divide 5 by 12 $(=4\times3,)$ is the ſame as to divide it firſt by 4, (which gives a terminate Quotient, extending to two places of Decimal Fractions, $\frac{5}{4}=1,25$;) and then to divide this Quotient by 3, $(\frac{1,25}{3,00}=0,41666,$ &c.) Which Diviſion by 3, doth

therefore

therefore not operate singly, till the 3d Place of Decimal Fractions; when all the significant Figures, of the first Quotient are spent. So $\frac{2}{15}=0,13333$ &c. that is (because of $15=5\times3$,) $\frac{2}{5}=0,4$. And $\frac{0,4}{3,0}=0,13333$ &c. And $\frac{45}{56}=0,803571428571428$ &c. That is, (because of $56=8\times7$;) $\frac{45}{8}=5,625$ and $\frac{5,625}{7,000}=0,80357142857142 8$ &c. And the like in other Cases.

I have insisted the more particularly on this, (says the Dr.) because I do not remember that I have found it so considered by any other.

But the Concinnity (continues the Dr.) which thus appears in the interminate Quotient of a Division, (the same Numbers again returning in a continual Circulation;) is not to be expected in like Manner in the Extraction of Roots, (square, cubic, or of higher Powers.) For though the surd Root may be continued by Approximation in Decimal Parts, infinitely: Yet we have not therein the like Recurrence of the numeral Figures in the same Order, as in Division we had.

As $\sqrt{2}=1,41421356+$. Which yet hinders not but that this Approximation may be safely admitted in practice; and if so supposed infinitely continued, must be supposed to equal the Root of that surd Number; as truly as $0,33333$ &c. infinitely, to equal $\frac{1}{3}$.

Thus ends, on this Subject, as great a Mathematician as any in his Time. Upon whose Observations, I doubt not, Men began to speculate, and at last to contrive a Method (unthought of then) how to apply the Doctrine of Circulates to Arithmetical Operations.

N. B.

N. B. When I come to treat in this Book concerning Involution, and Evolution, or the Extraction of Roots, whether of ſquare, cubic, or of higher Powers, I ſhall there exhibit Examples, ſeemingly irrational or ſurd Numbers, which will have, in their Roots, the ſame Numbers again returning in a continual Circulation, as appears in the interminate Quotient of a Diviſion. And in that Place I ſhall alſo make it evident, that an infinite Number of ſuch Examples might be produced; though I am very ſenſible that it may be but ſeldom that any ſuch Examples ſhould occur in Practice, yet whenever any ſuch ſhall happen, its Root will then conſiſt of a mathematical exact Anſwer, and be as correct an Expreſſion, as is the Root of any rational Number whatever.

The next Author which came to my Hands, and occaſionally treats on the ſame Subject with Dr. *Wallis*, was the Ingenious Mr. *Jones*, who in his *Synopſis Palmariorum Matheſeos*, publiſhed in the Year 1706, pag. 104, 105, recites conciſely, and that but a Part too, of what the Dr. had largely explained before.

And ſince the above Gentleman, Mr. *Ward*, in his *Young Mathematician's Guide*, page 69, and other Authors, content themſelves only with informing their Readers that ſome Numbers will circulate, but none of them ſo much as intimate a Poſſibility of applying ſuch circulating Numbers to any arithmetical Operations, but by way of Approximation only.

The firſt Author on the following Subject, (ſo far as I can learn) who appear'd in Public, and applied circulating Numbers to arithmetical Operations, was the Reverend Mr. *Brown*, in his *Syſtem of Decimal Arithmetic*, publiſhed in the Year   I ſuppoſe about the Years 1708, or 1709: But I muſt leave its true Date for others to fill up; for after a very careful Enquiry in various Parts of this Kingdom, and a long Expectation for a Sight of that Book, I could not be ſo happy as to obtain it. Where-

fore

fore my Readers must be content with such Informations of this Author's Performance as I shall transcribe from Mr. *Cunn*, and from Mr. *Malcolm*.

The former Gentleman, in his Preface, observes, that though this Method of using Fractions is absolutely necessary to be known, yet no Treatise hitherto extant hath sufficiently handled it: And remarks this, That the Reverend Mr. *Brown*, in his *System of Decimal Arithmetic*, manages such interminate Decimals as have a single Digit continually repeated; but in Multiplication useth only such Factors as will produce a single Repetend in the Product, (being, as I suppose, continues Mr. *Cunn*, unwilling so much as to mention compound Repetitions) and in Division leaves the Practitioner to work without Exactness. Vide *Cunn's Preface*, page 5, 6.

The latter Gentleman, in his Preface, remarks, That Dr. *Wallis* is probably the first, as he has himself observed, who has distinctly considered this curious Subject of circulating Decimals. He has (says Mr. *Malcolm*) given us the fundamental Theory of it, but without Demonstration; nor has he meddled with the practical Part, or Way of managing infinite Decimals in arithmetical Operations. And Mr. *Brown*, in his Decimal Arithmetic, has handled but one single Case of the Practice, and that not compleatly neither. Vide *Malcolm's Preface*, page 11.

However, in my Opinion this first, though little, Intimation towards the Arithmetic of circulating Numbers, brings no small Reputation to that Reverend Gentleman; for probably it was his Performance, that set others upon thinking how to apply it more universally.

The first Book that came to my Hands, which treats of the Arithmetic of Circulates, is a Treatise wrote by the Ingenious Mr. *Cunn*, entitled, *A new and compleat Treatise of the Doctrine of Fractions*, first published in the Year 1714. Wherein he hath exhibited the Arithmetic of many curious Examples, both in single and compound circulating Decimals,

Decimals, pure and mixt; being the firſt great Work on the Subject.

And had this Gentleman done, what in his Preface he ſaid he firſt deſigned to do, *viz.* to have given Demonſtrations to his Examples, I am perſuaded we ſhould then have had no new Book on this Subject very ſoon, or at leaſt had no occaſion for one: But, unhappy for the young Learner! He wrote very conciſely, and in a way not eaſily to be comprehended by any. Nay, the great Mr. *Malcolm*, in the Preface, page 11. to his (in my Opinion) incomparable *New Syſtem of Arithmetic*, publiſhed in the year 1730, does not ſtick to ſay, that Mr. *Cunn* has choſen to expreſs his Rules in ſuch a manner, as to ſet the Reaſon as far out of view as poſſible. And a little lower he adds, I muſt obſerve this further Effect of Mr. *Cunn*'s Way of delivering theſe Rules, That by themſelves one could never or very hardly be led into the Reaſon of them. However, in the ſame Preface, Mr. *Malcolm* acknowledges himſelf to be indebted to him for one or two uſeful Hints.

Indeed it muſt be allowed, that Mr. *Cunn* was every way qualified to have ſet his whole Subject in a clearer Light, as is evident from his many curious Examples; but what prevented him from doing it I cannot ſay.

And I frankly own it coſt me no little Pains, ſome Years ago, to diſcover the Reaſons of the ſeveral Methods made uſe of by this Gentleman.

I muſt not let Mr. *Cunn* paſs, as the ſole Improver of this valuable Subject; no: He is ſo ingenuous and grateful as to acknowledge, in his Preface, that he cannot forget his Friend Mr. *Robert Flavell*, Schoolmaſter in St. *Giles*'s *in the Fields*, whoſe Hints and particular Methods had contributed to his Diſcovery of the Nature and Laws of circulating Figures.

That the Memories of the above Gentlemen may be preſerved for theſe their joint Labours, and remain in high

Esteem among Mankind, as long as Numbers continue to be useful to the World, is the hearty Wish of one of their Admirers. For no Person, who is ignorant of the Arithmetic of Infinite Decimals, can be said to understand Decimal Arithmetic perfectly well; because without its Assistance the Result of his Operations must generally be imperfect, and the Error very considerable too, when he deals with large Numbers *. And if one did not daily see Improvements made in almost every Art and Science, one should be tempted to affirm, that now Decimal Arithmetic was brought to its utmost Perfection. But to proceed.

Mr. *Malcolm*'s System, above recited, was the next Book on this Subject, that came to my Hands; wherein that great Author treats the whole Doctrine of Infinite Decimals in a manner somewhat different from either of the Authors before-mentioned, which take in his own Words: " That in the Rules of Multiplication and Division, " (which are the more complex and difficult Parts) Mr. " *Cunn*'s Directions are not so easily followed; and are besides much harder for the Memory than the Method I " have chosen, which depends all upon the easy and natural Explication of one single Proposition; *viz.* the " finding the finite Value of (or Vulgar Fraction equal to) " any circulating Decimal: for though the Demonstrations are omitted, the Rule ought to be as simple and " easy as possible. But I must observe this further Effect " of Mr. *Cunn*'s Way of delivering these Rules, That " by themselves one could never, or very hardly, be led " into the Reason of them, nor consequently into the Way " I have chosen; so that it will be the more easily believed that the Rules I have given, are the Effect of " Speculations made upon this Subject, before I saw this " Book; which I mention for this Reason only, that I " may not be thought ungrateful to one whom I acknowledge the first Author (the first great Improver Mr. *Malcolm* should have said) " upon this Practice, from " whom therefore I might otherwise be supposed to have

* Vide *the Preface*.

" bor-

"borrowed or deduced all that I ſay; and yet I do ac-
"knowledge I owe him one or two uſeful Hints." Vide *Malcolm's Preface*, page 12.

Indeed we are greatly obliged to this Gentleman for ſetting that Propoſition, with many others on this Subject, in ſo clear a Light; but whoever will carefully look into Mr. *Cunn*'s Examples, both in Multiplication and Diviſion, will find that he well underſtood that Propoſition alſo, though he no where particularly remarks or explains it. I muſt confeſs 'tis hard, nay almoſt impoſſible, for a young Learner to find out why or wherefore Mr. *Cunn* made uſe of other adequate figural Expreſſions inſtead of thoſe he firſt propoſed. But I ſhall in ſome places in this Tract have an occaſion to make a few more Remarks on thoſe two laſt Gentlemen; therefore ſhall proceed now to the next Author who wrote on this Subject, *viz.*

The ingenious *Alexander Wright*, A. M. Writing-Maſter at *Aberdeen*; who likewiſe in his *Treatiſe of Fractions*, publiſhed in the Year 1734, treats of the Arithmetic of Infinite Decimals. But as he therein freely owns that he proceeds wholly on the Foot of Mr. *Malcolm*, I have nothing to remark on his Method; but do here adviſe the Editor of his Book, before it paſſes into a ſecond Edition, to be careful (for the Sake of the young Learner) to correct what is amiſs in *Chap.* xvii. which ought to be all new wrote except its Rule, the firſt Example and the laſt. The Book in general well anſwers its Title, *viz. A plain, eaſy, and compleat Syſtem of Practical Fractions, both Vulgar and Decimal*; and I know not if there be a better Book on the Subject of Fractions of its ſize.

The next Book on this Subject, which came to my Hands, was a very laborious and curious Performance of Mr. *Benjamin Martin* of *Chicheſter*, entitled, *A new, compleat, and univerſal Syſtem or Body of Decimal Arithmetic*, printed ſo late as 1735. Where the Author in his Preface, ſpeaking of his Management of Infinite Decimals, ſays, The Foundation on which I have built this Superſtructure

is Mr. *Cunn*'s ſmall, but learned, Treatiſe of the Doctrine of Decimal Circulating Numbers; and with this Remark. But that great Maſter having laid the Foundation deep, and in a great Meaſure out of the vulgar Ken, I thought it might be of Service to young Students, a little to diſcloſe and lay it more open to their View.

And I could heartily wiſh that Mr. *Martin* had therein been more copious on this Subject, for the Sake of the young Student. In my Opinion no body better qualified than himſelf, to have made it exceeding eaſy and familiar to every common Capacity; which had he done, in all probability this attempt of mine had remained ſtill in Obſcurity: In which, how well I have ſucceeded, I muſt leave to others to determine. Not but that I take this Gentleman's Book, altogether, to be far, very far, the beſt Syſtem of Decimals that ever was publiſhed, or perhaps that this Age can hope or expect to ſee; containing their Applications whether Arithmetically or Geometrically to all uſeful Knowledge to the beſt Advantage in the various Arts, Trades, and Buſineſs of Life: In ſhort, it is impoſſible to ſay too much in Commendation of this his truly new and curious Syſtem: its Contents will beſt be ſeen at the End of this Book where it is advertiſed.

The laſt Book which I have ſeen, that treats on this Subject, is *Mr.* William Pardon*'s new and compendious Syſtem of practical Arithmetic*, printed in the year 1738; wherein the Doctrines of whole Numbers and Fractions, both Vulgar and Decimal, are ſet in a clear Light, and fully explained; and in which he hath followed Mr. *Cunn*'s Method of managing Infinite Decimals, much after the manner with Mr. *Martin*, in the four primary Rules. It is a valuable Book, and I wiſh its ingenious Author a ſuitable Encouragement for his uncommon Pains. But I beg leave in this Place, with all due Reſpect to the Author, to point out to him an haſty or inaccurate Aſſertion in the Body of his Book, *page* 171. where treating of the Property of ſome vulgar Fractions, in their producing ſuch and ſuch repeating Decimals in their Quotients, he there affirms,

That

That others there are (referring to Vulgar Fractions) which only approximate and never circulate; and gives for Examples $\frac{63}{87}$, and $\frac{215}{318}$. Whereas every vulgar Fraction will turn out either a Finite Expression, or a pure or a mixt circulating Expression; as manifestly appears from Dr. *Wallis*'s Observations before recited. For though Mr. *Malcolm*, in page 467, says, Incertain Decimals are such whose Numerator goes on for ever, (goes on infinitely, says Mr. *Wright* in his Tract, page 136.) without a constant Circulation of the same Figure or Figures, yet Mr. *Malcolm* in the same Page very justly observes, That no Incertain Decimal can ever arise from any Finite assigned Fraction; and that when they do, as in some Cases, necessarily occur in Practice of Arithmetic, (*viz.* in the Extraction of Surd Roots;) there is then no possibility of supplying their Defects perfectly, so that we must be content to do it by way of Approximation.

However, it must be allowed, that when even the Quotient runs deep e're the Circulation ends, we must be content to take an Approximant Decimal, instead of the Circulate, to avoid much labour and trouble. For such as $\frac{63}{87}$, will give, $\dot{7}24137931034482758620689655\dot{1}$ infinitely repeated in its Quotient. And $\frac{215}{318}$, will give, $67\dot{6}$ $100628930\dot{8}\dot{1}$ where 7610062 &c. would infinitely repeat in its Quotient.

What induced me to take so much Notice of the above Assertion, did not proceed from a cavilling Disposition, (be that far from me) but in order to prevent any Mistakes that might arise from a young Student's imbibing wrong Principles; for I am persuaded that this Mistake of Mr. *Pardon*'s arose from an Oversight, and not from want of Judgment.

And laſtly, to conclude——The Reader might reaſonably expect that I ſhould in this Place give him ſome Account, 1ſt, What prevailed upon me to write a new Treatiſe on this Subject, after ſo many learned Authors. And 2dly, That I ſhould alſo inform him particularly wherein I differ from them in the Management thereof.

As to the firſt, I thought a Treatiſe wrote entirely by itſelf on the Subject, without being mixt with other arithmetical Rules, would be the more acceptable Book, eſpecially to ſuch Students who think they have Books enough by them already on the ſeveral common Branches of Arithmetic.

And as to the ſecond, In the firſt Place I flatter myſelf, and I hope not without ſome Foundation, that mine will ſerve as a Key to open all the ſeeming Difficulties that young Learners may meet with in any of the Authors who have gone before me on this Subject. In the ſecond Place, upon the whole, my Reader will meet with many things here, which are no where elſe to be found. And laſtly, I here aſſure my Reader, that I have made it my utmoſt Endeavour that my Book ſhould every way anſwer to its Title Page, which, how well I have performed, let my Book declare for me: 'Tis but a little one, yet I hope its Uſefulneſs will prove larger than its Bulk; ſuch as it is, 'tis heartily at your Service as well as its Author.

From my School in *Sarum*,
September 20. 1740.

JOHN MARSH.

DECI-

DECIMAL ARITHMETIC MADE PERFECT, &c.

CHAP. I.

WHAT an Infinite, or Circulating Decimal Expression is, hath been already shewn in the foregoing Introduction. And as there are many other Definitions and Propositions necessary to be known for the Management of these, and of all other kinds of circulating Expressions, before we can readily give their Sum, or Difference, or Product, or Quotient; I shall to each (as they occur in Point of Place, or Order) prefix the Figures 1. 2. 3. 4. 5. &c. The Use and Advantage of which will soon appear.

Definitions and Propositions.

1. The Figure, (or Figures) continually repeating in any Numerical Expression, is called a Repetend, or Circulate, (for they are synonimous Terms.) And the first Figure (or Figures) of either is called the Given Repetend, or Circulate.

2. Repetends are either Single or Compound.

3. A single Repetend is that, which consists of one Figure continually circulating: As 7777 &c. where 7 would repeat infinitely in the Quotient: Or 4444 &c. where 4 would repeat infinitely in the Quotient: Or 8888 &c. where 8 would repeat infinitely in the Quotient.

4. A compound Repetend is that, which consists of two or more places of Figures continually circulating: As 353535 &c. where 35 would repeat infinitely in the Quotient: Or 007007007 &c. where 007 would repeat infinitely in the Quotient: Or 1358713587 &c. where 13587 would repeat infinitely in the Quotient.

5. Repetends, or circulating Expressions in general are either pure or mixt.

6. Pure Repetends are such as have no significant Figure or Figures, but what belong to the Repetend, or have only a o or o's betwixt them and the Decimal Point. As 3,333 &c. Or 45,4545 &c. Or 486,486486, &c. Or 81,5815815 &c. where Integral Numbers are concerned. Or as ,3636 &c. Or ,370370 &c. Or ,1219512195 &c. Or ,0666 &c. Or ,007474 &c. Or ,00384615384615 &c. Or ,0303 &c. Or ,0036700367 &c. where Decimal Places only are concerned.

7. Mixt Repetends are such as have some significant Figure or Figures prefixt before the Circulation begins; the Examples of which are exhibited in the three following Cases.

CASE I.

Examples, Where are only Decimal Places before the Circulation begins;
Thus ,5333 &c. or ,263434 &c. or ,62057845784 &c.

CASE II.

Examples, Where are either Integral, or both Integral and Decimal Places, before the Circulation begins;
Thus, 36,777 &c. or 3,842842 &c.
Or 2,4777 &c. or 52,38444 &c. or 10,54735873358 &c. or 159,10695757 &c.

CASE III.

Examples, Where the Repetends begin in the Integral Part with Integrals before them.

Thus 57,777 *&c.* or 329,494 *&c.* or 6547,4747 *&c.* or 87444,444 *&c.*

8. Every Mixt Repetend, or Circulate, consists of two Parts, *viz.* a Finite Part and a Circulating Part:

As in the Examples of *Case* I. preceding. There the ,5 the ,26 and the ,620 which are not concerned to make or form the Circulate, are the Finite Parts of their several Repetends; and their several Circulating Parts are ,03 ,0034 ,0005784. Both which Parts are most commodiously distinguished after the following manner; *viz.*

5 -|- 03
26 -|- 0034
620 -|- 0005784

Observe to prefix as many o's before the Given Circulate, as there are Places of Figures betwixt it and the Decimal Point. The Reason is manifest.

For ,5 -|- ,03 = ,53; and ,26 -|- ,0034 = ,2634 *&c.*

(2.)

Again, in the Examples of *Case* II. preceding: There the 36, the 3, the 2,4 the 52,38 the 10,54 and the 159,1069, are the Finite Parts of their several Repetends: And their several Circulating Parts are ,7 ,842 ,07 ,004 ,007358 and ,000057.

Both which Parts are most commodiously distinguished after the following manner; *viz.*

36 -|- 7 3 -|- 842	} Here being no Decimal Places before their Circulations begin, therefore there are no o's prefixt to their given Circulating Parts.
24 | 07	
5238 | 004	
1054 -|- 007358	
1591069 -|- 000057	

D *Note*,

Note, The Number of o's prefixt to each Given Circulate, most fitly shew the Number of Decimal Places in their several Finite Parts: As the Expression 620 + 0005784 denote ,620 + ,0005784 = ,6205784. And 24 + 07 denote 2,4 + ,07 = 2,47. And 1591069 + 000057 denote 159,1069 + ,000057 = 159,106957 *&c.*

(3.)

And lastly, in the Examples of *Case* III. preceding: Their Finite and Circulating Parts are most commodiously distinguished after the following manner; *viz.*

As	57,77 *&c.* is	distinguished thus	50 + 70
	329,494 *&c.*	thus	320 + 940
	6547,47 *&c.*	thus	6500 + 4700
	57945,945 *&c.*	thus	57000 + 945000.

Here 50. 320. 6500. 57000. the Integral Numbers, not concerned to make or form their Circulates, are their several Finite Parts; and their several Circulating Parts are 7,0. 9,40. 47,00. 945,000.

Note, That in Examples of this last Form, where the Repetend begins in the Integral Part, with Integrals before it, care must be taken that we annex as many o's to each Given Single or Compound Repetend, as there are Integral Places concerned in the Circulating Part preceding the Decimal Point.

The Necessity of rightly distinguishing the Finite and Circulating Parts, as above, of any Mixt Circulate, will appear by and by.

I am very sensible that the Repetends, which may occur in the last Form, might by *Transformation* be made to begin next the Decimal Point; and then their Finite and Circulating Parts would be distinguished after the manner with the two first Examples in *Case* II. above. But I would not perplex the young Learner with too many Rules together, therefore I omit it.

And

And to avoid the Trouble for the future of writing down the Given Repetend or Circulate, whether Single or Compound, more than once (except ſometimes for Illuſtration ſake) we ſhall henceforward diſtinguiſh each by placing a Period over the firſt Figure, or over the firſt and laſt Figures of the given Repetend.

As the Expreſſion 7777 &c. will be diſtinguiſhed thus $\dot{7}$; and the Expreſſion 4444 &c. thus $\dot{4}$; and 8888 &c. thus $\dot{8}$; as ſingle Repetends.

And the Expreſſion 353535 &c. thus $\dot{3}\dot{5}$; and 007007 &c. thus $\dot{0}0\dot{7}$; and 1358713587 &c. thus $\dot{1}358\dot{7}$; as Compound Repetends. Proceed we now,

9. To find the Finite Value of any Circulating Expreſſion :

O R,

How to find a Vulgar Fraction equivalent to any Repetend or Circulate, whether Single or Compound, Pure or Mixt.

C A S E I.

Of Pure Circulates.

R U L E.

When the Expreſſion is a Decimal Pure Circulate, then it is equal to a Vulgar Fraction whoſe Numerator is the given Circulate ; and its Denominator will be as many 9's as there are Places of Figures in the given Circulate, with as many o's annext as there happen to be o's betwixt it and its Decimal Point.

Examples.

(1) $,\dot{3}=\frac{3}{9}$. (2) $,\dot{5}=\frac{5}{9}$. (3) $0\dot{5}=\frac{5}{90}$. (4) $,00\dot{8}=\frac{8}{900}$. (5) $,000\dot{4}=\frac{4}{9000}$. And (6) $,\dot{5}\dot{7}=\frac{57}{99}$. (7) $,\dot{3}4\dot{5}=\frac{345}{999}$. (8) $,0\dot{5}\dot{3}=\frac{53}{990}$. (9) $00\dot{5}\dot{3}=\frac{53}{9900}$. (10) $,0\dot{7}4\dot{5}=\frac{745}{9990}$. (11) $,00\dot{8}75\dot{4}=\frac{8754}{999900}$. (12) $,0000\dot{7}5843\dot{6}=\frac{758436}{9999990000}$. And (13) $,\dot{0}\dot{3}=\frac{3}{99}$. (14) $,00\dot{3}36\dot{7}=\frac{3367}{999999}$. (15) $\dot{0}00\dot{1}=\frac{1}{9999}$.

CASE II.

Of Pure Circulates.

RULE.

When the Expreſſion is a Pure Circulate conſiſting of Integral Figures, then its Finite Value, or Equivalent Vulgar Fraction, is found by making its Numerator to be the Given Circulate, with as many o's annext to it, as there are Integral Places of Figures in the Given Circulate; and its Denominator will be as many 9's as the Circulate hath Places of Figures.

Examples.

(1) $\dot{7},\dot{5}=\frac{750}{99}$. (2) $\dot{3},\dot{4}=\frac{340}{99}$. (3) $\dot{3}\dot{4},34=\frac{3400}{99}$. (4) $\dot{3}\dot{4}3,4=\frac{34000}{99}$. (5) $\dot{4}6,5\dot{8}=\frac{465800}{9999}$. (6) $\dot{3}4\dot{7},347=\frac{347000}{999}$. (7) $\dot{1}358,\dot{7}13587=\frac{135870000}{99999}$. (8) $\dot{4}357\dot{8}2,435782=\frac{435782000000}{999999}$.

N. B.

N. B. When the Circulating Expreſſions, which may occur in Practice, are like theſe following, *viz.* $\dot{1}219\dot{5}1,2195$ or $\dot{1}219\dot{5}12,195$ or $\dot{1}219\dot{5}121,95$ then their Equivalent Vulgar Fractions are found by having for their ſeveral Denominators as many 9's, as aboveſaid; and their Numerators muſt be the Circulate itſelf, with as many 0's annexed, as there are Integral Places of Figures in the whole Integral Number.

$$\text{As } \dot{1}219\dot{5}1,2195 = \frac{12195000000}{99999}$$

$$\text{And } \dot{1}219\dot{5}12,195 = \frac{121950000000}{99999}$$

$$\text{And } \dot{1}219\dot{5}121,95 = \frac{1219500000000}{99999} \text{ and ſo on.}$$

CASE I.

Of Mixt Circulates.

RULE.

When the Expreſſion is a Mixt Circulate, whoſe Repetend conſiſts of Decimal Places only, then its Finite Value, or Equivalent Vulgar Fraction, is found thus:

Firſt ſet down its Finite Part, (found by *Article* 8.) and multiply it by as many 9's, as there are Places of Figures in the Given Circulate; to the Product add its Circulating Part, and that Sum ſhall be the Numerator of the required Fraction. And for its Denominator, take the Denominator of the Circulating Part of the Repetend (found by *Caſe* I. of Pure Circulates.) and this Fraction will be its Equivalent Single Fraction.

Ex. (1) $,2\dot{7} = \frac{2\times9+07}{90} = \frac{25}{90}$ its E. S. F.

Ex.

Ex. (2) $,4\dot{7}\dot{5} = \frac{4 \times 99 + 075}{990} = \frac{471}{990}$ its E. S. F.

Ex. (3) $,348\dot{5}73\dot{4} = \frac{348 \times 9999 + 0005734}{9999000} =$
$\frac{3485386}{9999000}$ its E. S. F.

Ex. (4) $36,\dot{7} = \frac{36 \times 9 + 7}{9} = \frac{331}{9}$ its E. S. F. *i. e.* Equivalent Single Fraction.

Ex. (5) $3,\dot{8}4\dot{2} = \frac{3 \times 999 + 842}{999} = \frac{3839}{999}$ its E. S. F.

Ex. (6) $20,0\dot{4}\dot{6} = \frac{200 \times 99 + 046}{990} = \frac{19846}{990}$ its E.S.F

Ex. (7) $8,327\dot{4}6\dot{1} = \frac{8327 \times 999 + 000461}{999000} = \frac{8319134}{999000}$
its E. S. F.

CASE II.

Of Mixt Circulates.

RULE.

When the Expreffion is a Mixt Circulate, which begins in fome Integral Place, then its Finite Value, or Equivalent Vulgar Fraction, is found in this manner: Firft fet down its Finite Part, (found by *Art.* 8.) and multiply it by as many 9's, as there are Places of Figures in the Given Repetend; to the Product of which add its Circulating Part, (found as above) and that Sum fhall be the Num^r^. of the required Fraction. And for its Denom^r^. take as many

many 9's, as the Given Circulate hath Places of Figures; and this Fraction ſhall be its Equivalent Single Fraction.

Ex. (1) $5\dot{7},77$ *&c.* $= \frac{50\times9+70}{9} = \frac{520}{9}$ its E. S. F.

Ex. (2) $3\dot{2}\dot{9},494$ *&c.* $= \frac{320\times99+940}{99} = \frac{32620}{99}$ its E. S. F.

Ex. (3) $42\dot{7}5,8\dot{4}7584$ *&c.* $= \frac{4200\times9999+758400}{9999}$ $= \frac{42754200}{9999}$ its E. S. F.

Ex. (4) $57\dot{9}4\dot{5},945$ *&c.* $= \frac{57000\times999+945000}{999} =$ $\frac{57888000}{999}$ its E. S. F.

Ex. (5) $87\dot{4}44,44$ *&c.* $= \frac{87000\times9+4000}{9} = \frac{787000}{9}$ its E. S. F.

Ex. (6) $57\dot{9}46\dot{7},946$ *&c.* $= \frac{500000\times9999+794600000}{9999}$ $= \frac{5794100000}{9999}$ its E. S. F.

In theſe and the foregoing Caſes I have exhibited Examples, by which all the Varieties that can, I think, poſſibly happen in Practice, may readily be reduced to their Equivalent Vulgar Fractions.

The

The Proof.

And to prove that the Equivalent Vulgar Fraction found, is equal to its Given Pure, or Mixt Circulate, you muſt divide its Num^r. by its Denom^r. and then if the Quotient turns out the Given Pure, or Mixt Circulate, you have a certain Proof that you have the exact Vulgar Expreſſion.

I am very ſenſible that many of the above-found Vulgar Fractions would reduce to lower Equivalent Expreſſions; but I choſe to ſet them down as they firſt offered themſelves, for fear the Operations ſhould appear too complex to a young Practitioner.

I ſhould now proceed to ſhew a more eaſy, as well as a more Expeditious Method, how to find the Equivalent Vulgar Fraction to any Mixt Circulate Expreſſion.

But previouſly to this, it is neceſſary that I here ſhew a Method.

10. 1ſt, How to multiply any Given Number by any Number of 9's in a very narrow Compaſs.

Example 1.

As, let 5674 be given to be multiplied by 99.

Operation.

Here 567400 is 100 times 5674

Therefore Subſt. 5674 the given Multiplicand, and it

will leave $561726 = 5674 \times 99$.

Example

Example 2.

Again, let 715892 be given to be multiplied by 9999.

Operation.

Here 7158920000 is 10000 times 715892
Therefore Subst. 715892 the given Multiplicand, and it

will leave 7158204108 = 715892×9999.

Example 3.

Let 475050 be given to be multiplied by 999999.

Operation.

Here 475050000000 is 1000000 times 475050
Therefore Subst. 475050 the given Multiplicand, and it

will leave 475059524950 = 475050×999999.

From the three preceding Examples it is very manifest, that to multiply any Numerical Expression by what Number of 9's you please, 'tis most readily done: 1st, By annexing as many 0's to the given Multiplicand, as the Multiplier consists of 9's, and then, from it thus Increased, substract the given Multiplicand, their Difference will be the exact Product required.

Observe, that if either, or both the Factors had been a Finite Decimal, or a Finite Mixt Expression, yet the Operation would have been the same; only then you must have mark'd off the Fractional Part in the Product, according to the Method of common Decimals.

(2dly,)

11. But let us suppose some Number were required to be added to the Product, where the Multiplier consists of 9's only.

Example 1.

Let 715892 be given to be multiplied by 99999, to whose Product it is required to add 5746: *Quere* the Number sought?

RULE.

In such a Case, when the Multiplicand is increased as before directed, it will be 71589200000: and to this increased Number add the Number required to be added, (which in this Example is 5746) and it will then become 71589205746: from which substract the given Multiplicand, and then its Difference will be the Number sought.

Operation.

Subst.	71589205746	the Multiplicand increased, as last directed;
	715892	the given Multiplicand;
Answ.	71588489854	the Number sought.

Example 2.

Let 875492 be given to be multiplied by 999999, to whose Product its required to add 896547: *Quere* the Number sought?

Operation.

Subst.	875492896547	the Multiplicand increased, &c.
	875492	the given Multiplicand:
Answ.	875492021055	the Number sought.

Example 3.

Let 57 be given to be multiplied by 99999, to whose Product it is required to add 49500: *Quere* the Number sought?

Operation.

Operation.

Subst.	5749500	the Multiplicand increased, &c.
	57	the given Multiplicand;
Answ.	5749443	the Number sought.

If these Examples are well understood by the young Learner, then what follows in the next Article will need no Illustrations. Therefore I shall proceed to shew,

12. A shorter Method, how to find the Finite Value, or the Equivalent Vulgar Fraction, of any Mixt Circulate or Repetend.

RULE.

From the Mixt Circulate substract the Figure or Figures, which stand before the first Figure of the Circulate, and Note its Difference for the Numerator, with the three following Observations.

1st, If the Circulate begins next below the Decimal Point, then its Difference, as above, is the Numerator; and its Denominator must be as many 9's as the Circulate consists of Places of Figures.

2dly, If the Circulate begins at any Place, not next, but any where else, below the Decimal Place, annex as many o's to the 9 or 9's as abovesaid for its Denominator, as are the Places of Figures between the Decimal Point and the first Figure of the Circulate. Its Numerator must be as above directed.

3dly, If the Circulate begins in the Integral Figures, then annex as many o's to its Numerator found, as above directed, as there are Integral Places between the first Figure of the given Circulate, inclusive to the Decimal Point; and its Denominator must be as many 9's as the Circulate consists of Places of Figures.

What hath been said above will be best apprehended by the following Examples, taken, for the most part, from the preceding Examples of Mixt Circulates, that the attentive Learner might easily see, by comparing both together, how much more expeditious this last Method is than that.

Examples for Observation 1.

(1) $5,\dot{7}$ Here $57-5=52$. Therefore $\frac{52}{9}$ is its E. S. F. (*i. e.*) Equivalent Single Fraction.

(2) $100,4\dot{7}$. Here $10047-100=9947$. Therefore $\frac{9947}{99}$ is its E. S. F.

(3) $347,\dot{5}8\dot{4}$. Here $347584-347=347237$. Therefore $\frac{347237}{999}$ is its E. S. F.

(4) $3,\dot{8}4\dot{2}$. Here $3842-3=3839$. Therefore $\frac{3839}{999}$ is its E. S. F.

(5) $27,\dot{5}475\dot{3}$. Here $2754753-27=2754753$. Therefore $\frac{2754753}{99999}$ is its E. S. F.

Examples for Observation 2.

(1) $,\dot{2}7$. Here $27-2=25$. Therefore $\frac{25}{90}$ is its E. S. F.

(2) $,\dot{4}\dot{7}5$. Here $475-4=471$. Therefore $\frac{471}{990}$ is its E. S. F.

(3)

(3) $,348\dot{5}73\dot{4}$. Here $3485734-348=3485386$. Therefore $\frac{3485386}{9999000}$ is its E. S. F.

(4) $20,0\dot{4}\dot{6}$. Here $20046-200=19846$. Therefore $\frac{19846}{990}$ is its E. S. F.

(5) $8,327\dot{4}6\dot{1}$. Here $8327461-8327=8319134$. Therefore $\frac{8319134}{999000}$ is its E. S. F.

(6) $3392,73993\dot{4}8\dot{8}$. Here $339273993488-339273993=338934719495$. Therefore $\frac{338934719495}{99900000}$ is its E. S. F.

Examples for Observation 3.

(1) $57,\dot{7}$. Here $57-5=52$. Therefore $\frac{520}{9}$ is its E.S.F.

(2) $32\dot{9},\dot{4}$. Here $3294-32=3262$. Therefore $\frac{32620}{99}$ is its E. S. F.

(3) $42\dot{7}5,84$. Here $427584-42=427542$. Therefore $\frac{42754200}{9999}$ is its E. S. F.

(4) $57\dot{9}4\dot{5},945$. Here $57945-57=57888$. Therefore $\frac{57888000}{999}$ is its E. S. F.

(5) $87\dot{4}44{,}4$. Here $874-87=787$. Therefore $\frac{787000}{9}$ is its E. S. F.

(6) $5\dot{7}94\dot{6}7{,}946$. Here $57946-5=57941$. Therefore $\frac{5794100000}{9999}$ is its E. S. F.

13. If the Repetend of any circulating Expreſſion is 9, then the Value, or Sum, of that Series of 9's is an Unit in the Place next that Repetend on the left Hand.

Thus $,\dot{9}=1$. And $,0\dot{9}=,1$. And $,00\dot{9}=,01$. And $,000\dot{9}=,001$ Or $9,\dot{9}=10$. And $19,\dot{9}=20$. And $399,\dot{9}=400$ and ſo on.

Demonſtration.

The Reaſon is manifeſt; for that $,9=\frac{9}{10}$ wants only $\frac{1}{10}$ of Unity; And $,99=\frac{99}{100}$ wants but $\frac{1}{100}$ of Unity; And $,999=\frac{999}{1000}$ wants but $\frac{1}{1000}$ of Unity; And $,9999=\frac{9999}{10000}$ wants but $\frac{1}{10000}$ of Unity, &c. So that if the Series of 9's were infinitely continued, the Difference between that Series of 9's and 1 would be equal to Unity divided by Infinity, that is, nothing at all.

14. If the Denominator of a Vulgar Fraction be 9, or 9's, having its Numerator leſſer than its Denominator, but conſiſting of the ſame Number of Places of Figures, as there are 9's in its Denominator; then its Numerator will become a Pure Circulate, whoſe firſt Figure will begin to circulate immediately after the Decimal Point.

Examples.

Examples.

As $\frac{8}{9} = ,\dot{8}$. And $\frac{1}{9} = ,\dot{1}$. And $\frac{57}{99} = ,\dot{5}\dot{7}$. And $\frac{587}{999} = ,\dot{5}8\dot{7}$. And $\frac{1058}{9999} = ,\dot{1}05\dot{8}$. And $\frac{26829}{99999} = ,\dot{2}682\dot{9}$ and so on. Or as $\frac{40}{99} = ,\dot{4}\dot{0}$. And $\frac{500}{999} = ,\dot{5}0\dot{0}$. And $\frac{100000}{999999} = ,\dot{1}0000\dot{0}$ and ſo on.

15. If the Denominator of a Vulgar Fraction conſiſt of 9's, not having the ſame Number of Places of Figures in its Numerator, as there are 9's in its Denominator, then its Numerator, with as many 0's ſet before it, as is the Exceſs of the 9's in the Denominator, more than the Places of Figures in its Numerator, will be a Pure Circulate; which will alſo begin to circulate immediately after the Decimal Point.

Examples.

As $\frac{8}{99} = ,\dot{0}\dot{8}$. And $\frac{1}{999} = ,\dot{0}0\dot{1}$. And $\frac{5}{9999} = ,\dot{0}00\dot{5}$. And $\frac{7}{999999} = ,\dot{0}0000\dot{7}$. And $\frac{74}{999} = ,\dot{0}7\dot{4}$ And $\frac{157}{99999} = ,\dot{0}015\dot{7}$. And $\frac{3408}{999999} = ,\dot{0}0340\dot{8}$ and ſo on.

16. If the Denominator of a Vulgar Fraction conſiſt of 9's with 0's, not having the ſame Number of Places of Figures in its Numerator, as there are 9's in its Denominator; then its Numerator, with as many 0's ſet before it, as is the Exceſs of the 9's in the Denominator more than the Places of Figures in its Numerator, will alſo be a Pure Circulate; but will not begin to circulate, until as many 0's are ſet between it and the Decimal Point, as there are 0's found in the Denominator.

Examples.

Examples.

As $\frac{7}{990} = ,0\dot{0}\dot{7}$. And $\frac{8}{9990} =, 0\dot{0}0\dot{8}$. And $\frac{6}{99000}$ $= ,000\dot{0}\dot{6}$. And $\frac{35}{9990} = ,0\dot{0}3\dot{5}$. And $\frac{457}{99999000} =$ $,000\dot{0}047\dot{5}$. And $\frac{75843}{9999990000} = ,0000\dot{0}7584\dot{3}$ and ſo on.

Note, The Expreſſion $\frac{7}{90} = ,0\dot{7}$; And $\frac{8}{9000} = ,000\dot{8}$.

Now what hath been affirmed in the laſt three preceding Articles, might all be eaſily proved by Diviſion in common Decimals only.

There are many other curious Remarks which might be made on this Subject concerning the Properties of Vulgar Fractions; which an attentive Learner will eaſily diſcover, if he carefully reflects on the foregoing Examples, together with what I have tranſcribed in my Introduction, from the learned Dr. *Wallis*; therefore I ſhall not proceed any farther therein, but ſhall haſten to the next Article.

17. A ſingle Repetend might be made as a Pure Compound Repetend. Thus $,\dot{4}$ might be made $,\dot{4}\dot{4}$ or $,\dot{4}4\dot{4}$ or $,\dot{4}44\dot{4}$ or $,\dot{4}444\dot{4}$ and ſo on.

Or it might be made a Mixt Repetend. Thus $,\dot{4}$ might be made $,4\dot{4}$ or $,44\dot{4}$ Or $,44\dot{4}\dot{4}$ or $,44444\dot{4}\dot{4}$ and ſo on, as Neceſſity may require.

In fine, any given Repetend, whether Single or Compound, either Pure or Mixt, may be transformed, or changed, into another Repetend, conſiſting of the ſame Number of Places, or of a greater Number of Places of Figures

Figures at pleasure; and yet, still each New Expression will retain the same Value with its first given circulating Expression.

RULE.

Write down the Given Repetend as often as is necessary for its Transformation; and then mark off for a New Repetend as many Places of Figures, as are required.

Thus the Expression $,\dot{4}$ might be transformed into $,\dot{4}\dot{4}$ or $,\dot{4}4\dot{4}$ *&c.* as above.

And the Expression $,\dot{5}\dot{7}$, might be transformed into $,\dot{5}75\dot{7}$ or into $,\dot{5}7575\dot{7}$ or into $,\dot{5}75757\dot{5}\dot{7}$ *&c.*

OR it might be transformed into $,5\dot{7}\dot{5}$ or into $,57\dot{5}\dot{7}$ or into $,575\dot{7}\dot{5}$ or into $,5757\dot{5}7575\dot{7}$ and so on, as Necessity may require.

And the Expression $,38\dot{6}\dot{3}$ might be transformed into $,38\dot{6}36\dot{3}$; or into $,38\dot{6}3636\dot{3}$; or into $,3863\dot{6}36\dot{3}$; or into $,386363\dot{6}\dot{3}$; or into $,38636\dot{3}6363\dot{6}$ *&c.*

It is very probable that some of my Readers may not easily perceive that the Circulating Expression $,\dot{4}$ is of the same Value with, or equal to $,\dot{4}\dot{4}$; or to $,\dot{4}4\dot{4}$; or to $,4\dot{4}\dot{4}$ *&c.* as is above asserted. Therefore, for their better Apprehension, I shall here demonstrate,

(1st,) That the Expression $,\dot{4}=,\dot{4}\dot{4}=,\dot{4}4\dot{4}=,\dot{4}44\dot{4}=,\dot{4}444\dot{4}=,\dot{4}4444\dot{4}$ *&c.*

F As

As alſo that the Expreſſion $,\dot{5}\dot{7} = ,\dot{5}75\dot{7} = ,\dot{5}7575\dot{7} =$ $,\dot{5}757575\dot{7}$ *&c.*

L E M M A.

When two Fractions of different Expreſſions are equal to each other, as $\frac{3}{7} = \frac{12}{28}$, then the Numerator of the one Expreſſion is to the Numerator of the other Expreſſion, as the Denominator of that is to the Denominator of this. Or in other words, the Numerator of the one Expreſſion is to its Denominator, as the Numerator of the other Expreſſion is to its Denominator.

For Example, If $\frac{3}{7}$ are equal to $\frac{12}{28}$, then 3 is to 12, as 7 is to 28; or 3 is to 7, as 12 is to 28.

And when four Numbers are thus Proportional, then the Product of the Means is equal to the Product of the Extreams.

For $12 \times 7 = 84$ the Product of the Means.
And $3 \times 28 = 84$ the Product of the Extreams.

Wherefore when two Fractions of different Expreſſions are equal, the Products of the Numerator of the one by the Denominator of the other alternately will alſo be equal.

Hence then by this Method we might ſooner, and with more eaſe, determine, whether any two, or more, Fractions of different Expreſſions are equal to each other, than we can by the common Method of ſometimes many and very tedious Diviſions.

I come now to prove that $,\dot{4} = ,\dot{4}\dot{4} = ,\dot{4}4\dot{4}$ *&c.* That is (by *Art.* 9.) I am to prove that the Expreſſions $\frac{4}{9} = \frac{44}{99} = \frac{444}{999} = \frac{4444}{9999} = \frac{44444}{99999} = \frac{444444}{999999}$ *&c.* (*i. e.*) are

equal

equal to each other, and conſequently equal among themſelves.

Demonſtration.

1ſt, Becauſe $4\times99=44\times9=396$
And $4\times999=444\times9=3996$
And $4\times9999=4444\times9=39996$
And $4\times99999=44444\times9=399996$
And $4\times999999=444444\times9=3999996$.

Now, foraſmuch as the Products of the Numerator of the one Expreſſion by the Denominator of the other alternately are equal, when compared with each other, therefore it is manifeſt that the ſeveral different Expreſſions above are equal in Value each one to each other, and conſequently equal among themſelves. Which was to be demonſtrated.

The like Method of Demonſtration will alſo prove that the Expreſſion $,\dot{5}\dot{7}=,\dot{5}75\dot{7}=,\dot{5}7575\dot{7}=,\dot{5}757575\dot{7}$ *&c.* or however thus varied.

That is that $\frac{57}{99}=\frac{5757}{9999}=\frac{575757}{999999}=\frac{57575757}{99999999}$ *&c.*

For $57\times9999=5757\times99$. And $57\times999999=575757\times99$ *&c.*

2dly,

It remains that I alſo prove that the Expreſſion $,\dot{4}=,4\dot{4}=,44\dot{4}=,444\dot{4}$ *&c.* Or equal to $,4\dot{4}\dot{4}=,44\dot{4}\dot{4}=,4444\dot{4}4\dot{4}=,444\dot{4}444\dot{4}$ *&c.* or however thus varied.

So likewiſe that the Expreſſion $,\dot{5}\dot{7}=,5\dot{7}\dot{5}=,57\dot{5}\dot{7}=,575\dot{7}\dot{5}=,5757\dot{5}7575\dot{7}$ *&c.* or however thus varied.

 Demon-

Demonſtration.

Now (by *Art.* 9.) the Expreſſion $,\dot{4} = \frac{4}{9}$ its E. S. F.

$$\text{And } ,4\dot{4} = \frac{40}{90} = \frac{4}{9}$$

$$\text{And } ,44\dot{4} = \frac{400}{900} = \frac{4}{9}$$

$$\text{And } ,444\dot{4} = \frac{4000}{9000} = \frac{4}{9}$$

And where two, or more Expreſſions are equal to one and the ſame Expreſſion, they muſt be equal each one to each other, conſequently equal among themſelves.

Therefore $,\dot{4} = ,4\dot{4} = ,44\dot{4} = ,444\dot{4}$ *&c.* which was to be demonſtrated.

Again, I ſay that $,\dot{4} = ,4\dot{4}\dot{4} = ,44\dot{4}\dot{4} = ,4444\dot{4}4\dot{4} =$ $,444\dot{4}444\dot{4}$ *&c.* or however thus varied.

Demonſtration.

Now the Expreſſions $,\dot{4} = \frac{4}{9}$ as before.

$$\text{And } ,4\dot{4}\dot{4} = \frac{440}{990} = \frac{4}{9}$$

$$\text{And } ,44\dot{4}\dot{4} = \frac{4400}{9900} = \frac{4}{9}$$

$$\text{And } ,444\dot{4}4\dot{4} = \frac{444000}{999000} = \frac{4}{9}$$

$$\text{And } ,444\dot{4}444\dot{4} = \frac{44444000}{99999000} = \frac{4}{9}$$

And

And where two or more Expreſſions are equal to one and the ſame Expreſſion &c.

Therefore $,\dot{4}=,4\dot{4}\dot{4}=,44\dot{4}\dot{4}=,444\dot{4}44\dot{4}=,44\dot{4}4444\dot{4}$ &c. which was to be demonſtrated.

3dly, And laſtly, I ſay that the Expreſſion $,\dot{5}\dot{7}=,5\dot{7}\dot{5}=$ $,\dot{5}\dot{7}57=,5\dot{7}\dot{5}75=,5757\dot{5}75\dot{7}57$ &c.or however thus varied.

Demonſtration.

Now (by *Art.* 9.) the Expreſſion $,\dot{5}\dot{7}=\frac{57}{99}$ its E.S.F.

$$\text{And } ,5\dot{7}\dot{5}=\frac{570}{990}=\frac{57}{99}$$

$$\text{And } ,57\dot{5}\dot{7}=\frac{5700}{9900}=\frac{57}{99}$$

$$\text{And } ,575\dot{7}\dot{5}=\frac{57000}{99000}=\frac{57}{99}$$

And where two or more Expreſſions are equal to one and the ſame Expreſſion, they muſt be equal each one to each other, conſequently equal among themſelves.

Therefore $,\dot{5}\dot{7}=,5\dot{7}\dot{5}=,57\dot{5}\dot{7}=,575\dot{7}\dot{5}=,575757\dot{5}75\dot{7}$ &c. which was to be demonſtrated.

From the foregoing Demonſtrations it is manifeſt,

1ſt, That any Given Repetend, or Circulate, might be transformed or changed into another Repetend, conſiſting of the ſame Number of Places, or of a greater Number of Places of Figures at pleaſure; and yet, notwithſtanding ſuch Transformation, each new Expreſſion will retain

retain the ſame Value with its firſt Given Circulating Expreſſion.

2dly, Hence we may learn too, that by ſuch Transformations any two, or more Given Repetends might be made to begin and end together, as Neceſſity may require. The Uſe of which will be ſeen in Addition and Subſtraction, &c. But to proceed,

18. Repetends are either Similar or Diſſimilar.

19. Similar or Like Repetends are ſuch, whoſe firſt Figures of their ſeveral Repetends do begin in the ſame Place, (whether before or after the Decimal Point) and do conſiſt of the ſame Number of Places of Figures.

20. Diſſimilar or Unlike Repetends are ſuch, whoſe firſt Figures of their ſeveral Repetends do not begin in the ſame Place, (whether before or after the Decimal Point) or do not conſiſt of the ſame Number of Places of Figures, although they do begin in the ſame Place.

21. How to transform two or more Diſſimilar Repetends to Similar Repetends.

RULE.

1ſt, Make all the Repetends to begin together, (by Transformation) where that One begins, which ſtands loweſt, either above or below the Decimal Point. 2dly, And then to make them all end together; let each of the Given Repetends conſiſt of as many Places of Figures, from the Place where they are all made to begin together, as is the leaſt Common Multiple of the ſeveral Numbers of Places found in all the Given Repetends.

N. B. It is moſt convenient to take their leaſt Common Multiple, in order to have their Sum, or Difference, expreſſed in or by the feweſt Number of Places of Figures.

Some

Some of my Readers (and perhaps many) may not know the Method of finding the least common Multiple to two, or more given Numbers; therefore, for their sakes, I shall in this Place give its Definition, with a Rule how to find it; because I would not put them to the Trouble of turning to another Book for it.

22. The least Common Multiple to two, or more Numbers, is the least Number, which being divided by either of the Given Numbers, will leave nothing remaining. Thus 24 is the least Common Multiple to 2 . 3 . 4 . 6 . 8 and 12.

23. How to find the least Common Multiple to two, or more Given Numbers.

RULE.

Set down the Given Numbers, as underneath; then divide them severally by 2 or 3 *&c.* and their Quotes by the same, or any other Number; and their Quotes, if not Units already by the same, or any other Number; thus continuing, 'till Units only are their several Quotes, reserving their several Divisors; which Divisors multiply into each other, and their last Product shall be the Number sought; which will be the least common Multiple to the Given Numbers.

Example 1.

Find the least Common Multiple to 2 . 3 and 4.

Divisors	2	2 . 3 . 4
	2	1 . 3 . 2
	3	3 . 1
		1

Divisors
2
2
4
3
12

Answ. 12 is their least Common Multiple.

N. B.

N. B. I take down the 3, as above, until I make the Divisor 3.

Example 2.

Find the least Common Multiple to 2 . 4 and 5.

			Divisors
Divisors	2	2 . 4 . 5	2
	2	1 . 2 . 5	2
	5	1 . 5	4
		1	5
			20

Answ. 20 is their least Common Multiple.

Example 3.

Find the least Common Multiple to 2 . 4 . 6 . 10 : 12 and 15.

			Divisors
Divisors	2	2 . 4 . 6 . 10 . 12 . 15	2
	2	1 . 2 . 3 . 5 . 6 . 15	2
	3	1 . 3 . 5 . 3 . 15	4
	5	1 . 5 . 1 . 5	3
		1 . 1	12
			5
			60

Answ. 60 is their least Common Multiple.

Here follow the Examples of Dissimilar Repetends transformed or changed into Similar ones.

Example

Example 1.

Diſſimilar	made Similar.	
$,\dot{4}$	$,44\dot{4}$	Here they all are made to begin together; where that One begins, which ſtands loweſt from the Decimal Point.
$,5\dot{7}$	$,57\dot{7}$	
$,08\dot{3}$	$,08\dot{3}$	

Example 2.

Diſſimilar	made Similar.	
$,\dot{7}$	$,\dot{7}\dot{7}$	Here 2 is their leaſt Common Multiple.
$,\dot{5}\dot{4}$	$,\dot{5}\dot{4}$	

Example 3.

Diſſimilar	made Similar.	
$,\dot{4}7\dot{5}$	$,47\dot{5}4754\dot{7}$	Here they are all made to begin together, as above directed; and 6 is their leaſt Common Multiple.
$,3\dot{2}\dot{4}$	$,32\dot{4}2424\dot{2}$	
$,\dot{5}\dot{9}$	$,59\dot{5}9595\dot{9}$	
$,32\dot{7}$	$,32\dot{7}7777\dot{7}$	
$,\dot{1}$	$,11\dot{1}1111\dot{1}$	

Example 4.

Diſſimilar	made Similar.
475,39	475,39
687,5	687,58
543,	543,54

Here they are all made to begin together, as above directed; and 3 is their leaſt Common Multiple.

Example 5.

Diſſimilar	made Similar.
57,	57,777777777777777
8,49	8,498498498498498
7,5647	7,564764764764764
,8035748	,803574857485748

Here 12 is their leaſt Common Multiple.

Example 6.

Diſſimilar	made Similar.
4,75837	4,7583777777777777777777
,50794	,5079407940794079407940 79
8,007598	8,0075980759807598075980 75
,58	,5858585858585858585858
5,047	5,0474747474747474747474

Here they are all made to begin together, as above directed; and 20 is their leaſt Common Multiple.

24. A Finite, or Determinate Number, or Decimal Expreſſion, may be made a Similar Circulate, by annexing as many o's to it, as the Given Repetends in the Example may require; that is, when the Finite Expreſſion reaches not ſo low as where that one Repetend begins, which ſtands loweſt in Order of Place. But be careful to obſerve, that when the Finite Expreſſion reaches as low, or lower than where that One Repetend begins, which ſtands loweſt in Order of Place, then the Repetends muſt all be made to begin together, immediately with the firſt o, annexed to the Finite Expreſſion; and muſt all be made to end together, as before directed. *Note*, In this Caſe there are 2 Varieties, which I ſhall exhibit among the Examples of Addition of Single and Compound Circulates.

CHAP. II.

Addition of Circulates or Repetends.

Rule for Single Circulates.

IF the Repetends given are Diſſimilar, make them all Similar, (by *Art.* 21.) 1ſt, Then, if the Example conſiſts of Single Circulates only, add up the Right-hand Column by 9's, and place the Overplus, if any, or, if nothing, a 0 at its bottom, for a Single Circulate; and carry One for every 9 found in that Column to the next Place; and add up the other Columns, if any, by 10's as uſual in Addition of Common Decimals, and the Figures ſubſcribed at bottom ſhall be the Total ſought.

Rule for Compound Circulates.

But 2dly, When the Example conſiſts of Compound Circulates add up all the Columns, when made Similar, by 10's, with this Caution, *viz.* to the Right-hand Column add as many Units as there are 10's, (mentally * found) in that Column where all the Circulates do or are made to begin together; and then the Figures ſubſcribed at the bottom of the Circulating Columns ſhall be the Circulate ſought; and add up the other Columns by 10's, as above directed.

Mr. Cunn's *Rule for Compound Repetends is thus:*

Make all (*i. e.* the Repetends) Conterminous, (the Author means Similar) and then add as in Common Decimals, only to the Right-hand Column add as many

* By mentally found, its deſign'd that you previouſly add up the ſeveral Columns, which are in the Places below where the Repetends do or are all made to begin together, in order to diſcover how many 10's may be found in that Column, by that mental Addition; elſe ſometimes you may carry 1 Unit ſhort of the Truth to the firſt Right-hand Column.

Units

Units as there are 10's in that Column where the Repetends all begin together; and then the Figures subscribed to the aforesaid Columns shall be the first and last of the Repetend.

Mr. *Malcolm* (in his Preface, *pag*. 12.) complains that this Rule of Mr. *Cunn*'s is insufficient for a general Rule; and says, that though it will bring out the true Answer in some Cases, yet it is not universally good for all Cases; and in *page* 479. he gives the following Rule.

Make them (*i. e.* the Repetends) all Similar, then take the Sum of the Repetends upon a separate Paper, and divide it by a Number consisting all of 9's, as many as the Number of Places in the Repetend; the Remainder of the Division is the Repetend of the Sum, to be set under the Figures added, with 0's on the left Hand, if it has not as many Places as the Repetends; the Quote is to be carried to the next Column, and the rest of the Addition done by the common Rules.

This Rule of Mr. *Malcolm*'s is universally good for all Cases, and so is that of mine too; and I think it easier for Practice, as giving less Trouble: for by adding up the Repetends with my Caution, as above directed, you mentally divide their Sum by as many 9's, as the Repetend consists of Places of Figures, *&c.*

And indeed I can see but two Examples, *viz.* the 4th in *page* 76, and the 3d in *page* 77, throughout Mr. *Cunn*'s Book, (it is the 2d Edition now lying before me) where he could possibly make a Mistake in adding them by following his own Rule: but as he happily avoided it in both, I am of Opinion that Mr. *Cunn* himself understood his own Rule in as strict a manner, as I have cautiously worded mine.

I was willing to exhibit Mr. *Malcolm*'s Method, that the Learner might chuse which he found most easy for Practice. And the Reason why we should divide their Sum by

as

as many 9's, as the Repetends conſiſts of Places of Figures, muſt plainly appear to every attentive Reader, who ſhall reflect on what hath been ſaid concerning the Method of finding an Equivalent Vulgar Fraction to any Circulating Expreſſion, &c. I ſhall now proceed to give Examples.

CASE I.

Of Similar Single Circulates.

Ex. (1.)	(2.)	(3.)	(4.)
$,\dot{3}$	$,\dot{5}$	$,9\dot{3}$	$,291\dot{6}$
$,\dot{2}$	$,\dot{7}$	$,7\dot{3}$	$,208\dot{3}$
$,\dot{1}$	$,\dot{8}$	$,2\dot{6}$	$,041\dot{6}$
Sum $\dot{6}$	$,\dot{4}$	$,0\dot{6}$	$,708\dot{3}$
	Sum $2,\dot{6}$	Sum $2,0\dot{0}$	Sum $1,250\dot{0} = 1,25$

I ſhall throughout Addition, except in the laſt Example, make uſe of ſuch Circulates, as offer themſelves in my Tables annexed; that the Learner might have a Proof of his Examples from the Principles of Vulgar Fractions, which will be ſo many uſeful, as well as eaſy, Illuſtrations to the Whole; and by which I ſhall have this peculiar Advantage, of making uſe of fewer Words, which otherwiſe would be unavoidably neceſſary, and ſo ſwell this Book beyond its intended Bulk.

Illuſtrations of the foregoing Examples.

In *Ex.* 1. Their Equivalent Vulgar Fractions are $\frac{3}{9}$ + $\frac{2}{9}$ + $\frac{1}{9}$ = $\frac{6}{9}$ = $,\dot{6}$

In

In *Ex.* 2. They are $\frac{5}{9}+\frac{7}{9}+\frac{8}{9}+\frac{4}{9}=\frac{24}{9}=2\frac{6}{9}$ $=2,\dot{6}$

In *Ex.* 3. They are $\frac{14}{15}+\frac{11}{15}+\frac{4}{15}+\frac{1}{15}=\frac{30}{15}=2$

In *Ex.* 4. They are $\frac{7}{24}+\frac{5}{24}+\frac{1}{24}+\frac{17}{24}=\frac{30}{24}=1\frac{1}{4}$ $=1,25$

CASE II.

Of Diſſimilar Single Circulates.

Ex. (1.)

Diſſimilar	made Similar.
$,\dot{6}$	$,666\dot{6}$
$,58\dot{3}$	$,583\dot{3}$
$,2\dot{6}$	$,266\dot{6}$
$,708\dot{3}$	$,708\dot{3}$
Sum	$2,225\dot{0}$

That is 2,225 compleat.

Ex. (2.)

Diſſimilar	made Similar.
$,74,58\dot{3}$	$74,583\dot{3}$
$9,4\dot{6}$	$9,466\dot{6}$
$0,291\dot{6}$	$0,291\dot{6}$
Sum	$84,341\dot{6}$

Ex. (3.)

Diſſimilar	made Similar.
$47,\dot{1}$	$47,1111\dot{1}$
$1,08\dot{3}$	$1,0833\dot{3}$
$6,1\dot{6}$	$6,1666\dot{6}$
$2,3541\dot{6}$	$2,3541\dot{6}$
Sum	$56,7152\dot{7}$

Ex.

Ex. (4.)

Dissimilar	made Similar.
$4,9791\dot{6}$	$4,9791\dot{6}$
$5,\dot{6}8$	$5,6888\dot{8}$
$4,1875$	$4,1875\dot{0}$
	Sum $14,8555\dot{5}$
	That is $14,8\dot{5}$

Here the Finite Expression 4,1875 reaches not so low, as where that One Repetend begins, which stands lowest in Order of Place; therefore that Repetend governs in this Example, where they must all begin together: which is the 1st Variety.

Ex. (5.)

Dissimilar	made Similar.
$17,4\dot{8}$	$17,48\dot{8}$
$4,05$	$4,05\dot{0}$
$0,\dot{1}$	$0,11\dot{1}$
	Sum $21,65\dot{0}$
	That is 21,65 compleat

Here the Finite Expression 4,05 reaches as low, as where that One Repetend begins, which stands lowest in Order of Place; therefore that Finite Expression governs in this Example, where they must all begin together: which I call the 2d Variety.

Ex. (6.)

Ex. (6.)

Dissimilar	made Similar.	
$5,3\dot{6}$	$5,3666\dot{6}$	Here the Finite Expression 5,5625 teaches lower, than where that One Repetend begins, which stands lowest in Order of Place; therefore that Finite Expression governs the Transformation: as in the last Example.
$7,91\dot{6}$	$7,9166\dot{6}$	
$\dot{3}3,3$	$33,3333\dot{3}$	
$1,5$	$1,5000\dot{0}$	
$5,5625$	$5,5625\dot{0}$	
	$53,6791\dot{6}$	

I have been the more careful to exhibit the last 3 Examples, where Finite Expressions are given to be added with Circulating Expressions, because I have not seen it so cautiously expressed in any Author before me; and also to prevent the Learner's committing any Mistake, when such Varieties shall occur in Practice.

Illustrations of the foregoing Examples.

In *Example* 1. Their Equivalent Vulgar Fractions are $\frac{2}{3} + \frac{7}{12} + \frac{4}{15} + \frac{17}{24} = \frac{28831}{12960} = 2,225$ compleat.

In *Ex.* 2. They are $\frac{7}{12} + \frac{7}{15} + \frac{7}{24} = \frac{5796}{4320} = 1,341\dot{6}$ to which add their Integral Numbers (*viz.*) $9 + 74$ and their Total will be $84,341\dot{6}$.

In *Ex.* 3. They are $\frac{1}{9} + \frac{1}{12} + \frac{1}{6} + \frac{17}{48} = \frac{22248}{31104} = ,\dot{7}1527$, to which add their Integral Numbers (*viz.*) $2 + 6 + 1 + 47$, and their Total will be $56,7152\dot{7}$.

In *Ex.* 4. They are $\frac{47}{48} + \frac{31}{45} + \frac{3}{16} = \frac{64128}{34560} = 1{,}8\dot{5}$, to which add their Integral Numbers (*viz.*) $4 + 5 + 4$, and their Total will be $14{,}8\dot{5}$.

In *Ex.* 5. They are $\frac{22}{45} + \frac{1}{20} + \frac{1}{9} = \frac{5265}{8100} = {,}65$ compleat, to which add their Integral Numbers (*viz.*) $4 + 17$, and their Total will be $21{,}65$.

In *Ex.* 6. They are $\frac{11}{30} + \frac{11}{12} + \frac{1}{3} + \frac{1}{2} + \frac{9}{16} = \frac{92592}{34560} = 2{,}6791\dot{6}$, to which add their Integral Numbers (*viz.*) $5 + 1 + 33 + 7 + 5$, and their Total will be $53{,}6791\dot{6}$.

Examples wherein are Compound Repetends or Circulates given to be added.

C A S E I.

Of Similar Compound Circulates.

Ex. (1.)	*Ex.* (2.)
$,\dot{5}7142\dot{8}$	$5{,}\dot{8}5714\dot{2}$
$,\dot{2}8571\dot{4}$	$20{,}\dot{7}6923\dot{0}$
$,\dot{1}4285\dot{7}$	$0{,}\dot{3}8095\dot{2}$
Sum $,\dot{9}9999\dot{9}$	Sum $27{,}\dot{0}0732\dot{6}$

That is 1 compleat.

Illustrations

Illustrations of the foregoing Examples.

In *Example* 1. Their Equivalent Vulgar Fractions are $\frac{4}{7}+\frac{2}{7}+\frac{1}{7}=\frac{7}{7}=1$.

In *Ex*. 2. They are $\frac{6}{7}+\frac{10}{13}+\frac{8}{21}=\frac{3836}{1911}=2\frac{14}{1911}$, to which add their Integral Numbers, *viz*. 20+5, and their Total will be $27\frac{14}{1911}=27{,}\dot{0}0732\dot{6}$.

C A S E II.

Of Dissimilar Compound Circulates.

Ex. (1.)

Dissimilar	made Similar.	
,15625	$,15625\dot{0}\dot{0}$	Here the Finite Expression governs the Transformation, as in the 2d Variety in Single Circulates.
$,\dot{2}\dot{1}$	$,21212\dot{1}\dot{2}$	
	Sum $,36837\dot{1}\dot{2}$	

Ex. (2.)

Dissimilar	made Similar.	
,625	$,625\dot{0}0000\dot{0}$	Here the Finite Expression governs, as in the last Example.
$,\dot{4}5\dot{9}$	$,459\dot{4}5945\dot{9}$	
$,\dot{4}6153\dot{8}$	$,461\dot{5}3846\dot{1}$	
	Sum $1,545\dot{9}9792\dot{0}$	

 Ex. (3.)

Ex. (3.)

Diſſimilar	made Similar.
$,\dot{7}$	$,\dot{7}7777\dot{7}$
$,\dot{4}\dot{5}$	$,\dot{4}5454\dot{5}$
$,\dot{8}1\dot{4}$	$,\dot{8}1481\dot{4}$
	Sum $2,\dot{0}4713\dot{8}$

Ex. (4.)

Diſſimilar	made Similar.
$\dot{1}6\dot{2},$	$162,1\dot{6}2162\dot{1}$
$2,9\dot{3}$	$2,9\dot{3}3333\dot{3}$
$\dot{1}\dot{7}2,$	$172,7\dot{2}7272\dot{7}$
$3,\dot{7}6923\dot{0}$	$3,7\dot{6}9230\dot{7}$
	Sum $341,5\dot{9}1998\dot{9}$

Ex. (5.)

Diſſimilar	made Similar.
$134,\dot{0}\dot{9}$	$134,09\dot{0}9090\dot{9}$
$97,2\dot{6}$	$97,26\dot{6}6666\dot{6}$
$9,08\dot{3}$	$9,08\dot{3}3333\dot{3}$
$1,5$	$1,50\dot{0}0000\dot{0}$
$0,\dot{8}1\dot{4}$	$0,81\dot{4}8148\dot{1}$
	Sum $242,75\dot{5}7239\dot{0}$

Here the Finite Expreſſion reaches not ſo low, &c. Therefore it is as the 1ſt Variety.

Illuſtrations

Illustrations of the foregoing Examples.

In *Example* 1. Their Equivalent Vulgar Fractions are $\frac{5}{32}+\frac{7}{33}=\frac{3899}{1056}=,36837\dot{1}\dot{2}$

In *Ex.* 2. They are $\frac{5}{8}+\frac{17}{37}+\frac{6}{13}=\frac{5949}{3848}=$ $1,545\dot{9}9792\dot{0}$.

In *Ex.* 3. They are $\frac{7}{9}+\frac{5}{11}+\frac{22}{27}=\frac{5472}{2673}=$ $2,\dot{0}4713\dot{8}$.

In *Ex.* 4. They are $\frac{6}{37}+\frac{14}{15}+\frac{8}{11}+\frac{10}{13}=\frac{205714}{79365}$ $=2,5\dot{9}1998\dot{9}$, to which add their Integral Numbers, *viz.* $3+172+2+162$, and their Total will be $341,5\dot{9}1998\dot{9}$.

In *Ex.* 5. They are $\frac{1}{11}+\frac{4}{15}+\frac{1}{12}+\frac{1}{2}+\frac{22}{27}$ $=\frac{187722}{106920}=1,75\dot{5}7239\dot{0}$, to which add their Integral Numbers, *viz.* $1+9+97+134$, and their Total will be $242,75\dot{5}7239\dot{0}$.

Example

Example 6.

Dissimilar	made Similar.
5475	5475,475475475
1428,7	1428,728728728
81,7	81,717171717
1,574	1,574747474
5,0008	5,000888888
	Sum 6992,497012285

I have set my Examples above as abstract Numbers; but the whole Operation would have been the same, had I applied them to Money, or Weights, to Time, or Measures, *&c.*

CHAP.

CHAP. III.

Subtraction of Circulates or Repetends.

RULE.

IF the Repetends of the given Minuend and Subtrahend are not Similar, make them ſuch. And in all Caſes obſerve this, That when the Repetend of the Subtrahend is greater than the Repetend of the Minuend, (when Similars) that you add 1, to the Right-hand Place of the Subtrahend; then proceed as in Subtraction of Common Decimals; and their Difference ſubſcribed ſhall be the Repetend of the Remainder.

I ſhall diſtinguiſh the Examples in this Rule into Caſes, as I did thoſe in Addition, for the more ready Apprehenſion of the Learner.

CASE I.

Of Similar Single Circulates.

Examples.

(1.)		(2.)	
Minuend	$7,541\dot{6}$	Minuend	$10,9\dot{3}$
Subtrahend	$1,458\dot{3}$	Subtrahend	$4,2\dot{6}$
Remainder	$6,083\dot{3}$	Remainder	$6,6\dot{6}$
That is	$6,08\dot{3}$	That is	$\dot{6}$,

(3.)

Minuend $7{,}1\dot{6}$

Subtrahend $3{,}2\dot{6}$ That is 3,9 compleat

Remainder $3{,}9\dot{0}$

Illustrations of the foregoing Examples.

Ex. 1. Expressed in Vulgar Fractions is $7\frac{13}{24} - 1\frac{11}{24} = 6\frac{1}{12} = 6{,}08\dot{3}$ And whose Repetend of its Subtrahend is lesser, than the Repetend of its Minuend.

Ex. 2. Is $10\frac{14}{15} - 4\frac{4}{15} = 6\frac{2}{3} = 6{,}6\dot{6}$. And whose Repetend of its Subtrahend is greater, than the Repetend of its Minuend, where 1 was added (as in the Rule directed) to its Subtrahend, before the Subduction began. The Reason of which in all such Cases is manifest. For do but imagine the $\dot{3}$ and $\dot{6}$ above continued infinitely below; then after the first Subduction of the Right-hand Figure 6 from its opposite 3, we must carry 1 to the next Left-hand Figure 6. Or to express myself in other Equivalent Words thus, take $\frac{6}{9}$ from $\frac{3}{9}$ I cannot, but take $\frac{6}{9}$ from $\frac{12}{9}$ and the Remainder will be $\frac{6}{9} = \dot{6}$, as above.

Ex. 3. Is $7\frac{1}{6} - 3\frac{4}{15} = 3\frac{9}{10} = 3{,}9$ compleat as above. For the Repetend of its Subtrahend is equal to the Repetend of its Minuend, then consequently the Repetend of their Remainder must be $\dot{0}$.

CASE

CASE II.

Of Dissimilar Single Circulates.

Examples.

(1.)

	Dissimilar	made Similar.
Minuend	$13,1458\dot{3}$	$13,1458\dot{3}$
Subtrahend	$7,5$	$7,5000\dot{0}$
	Remainder	$5,6458\dot{3}$

(2.)

	Dissimilar	made Similar.
Minuend	$11,8125$	$11,8125\dot{0}$
Subtrahend	$2,\dot{7}$	$2,7777\dot{7}$
	Remainder	$9,0347\dot{2}$

(3.)

	Dissimilar lb	made Similar.
From	$5,0\dot{3}$	$5,033\dot{3}$
Take	$3,041\dot{6}$	$3,041\dot{6}$
	Remainder	$1,991\dot{6}$

(4.)

	Dissimilar	made Similar.
	Tuns	Tuns
From	$110,\dot{6}$	$110,6666\dot{6}$
Take	$94,1458\dot{3}$	$94,1458\dot{3}$
	Rem.	$16,5208\dot{3}$

Illustrations of the foregoing Examples.

Ex. 1. Expressed in Vulgar Fractions is $13\frac{7}{48} - 7\frac{1}{2} = 5\frac{31}{48} = 5,6458\dot{3}$.

Ex. 2. Is $11\frac{13}{16} - 2\frac{7}{9} = 9\frac{5}{144} = 9,0347\dot{2}$.

Ex. 3. Is from ℔ 5 : 0 : 8 Take ℔ 3 : 0 : 10 Remainder ℔ 1 : 19 : 10 = ℔ $1,991\dot{6}$.

Ex. 4. Is from 110 Tuns 13 C.wt. 1 Qr. 9 ℔ $5\frac{1}{3}$ Ozs. Take 94 Tuns 2 C.wt. 3 Qrs. 18 ℔ $10\frac{2}{3}$ Ozs. Remainder 16 Tuns 10 Cwt. 1 Qr. 18 ℔ $10\frac{2}{3}$ Ozs. $= 16,5208\dot{3}$ Tuns.

Examples

Examples wherein are Compound Repetends or Circulates to be subtracted.

CASE I.

Of Similar Compound Circulates.

Examples.

(1.)

From	$3,\dot{8}\dot{1}$
Take	$1,\dot{1}\dot{8}$
Rem.	$2,\dot{6}\dot{3}$

(2.)

From	$1,\dot{3}57142\dot{8}$
Take	$,\dot{7}85714\dot{2}$
Rem.	$,\dot{5}71428\dot{5}$
Or	$,\dot{5}7142\dot{8}$

Illustrations of the foregoing Examples.

Ex. 1. Expressed in Vulgar Fractions is $3\frac{9}{11} - 1\frac{2}{11} = 2\frac{7}{11} = 2,\dot{6}\dot{3}$.

Ex. 2. Is $1\frac{5}{14} - \frac{11}{14} = \frac{6}{7} = ,\dot{5}7142\dot{8}$.

CASE II.

Of Dissimilar Compound Circulates.

(1.)

	Dissimilar	made Similar.
	Cwt.	
From	$6\ ,\dot{5}7142\dot{8}$	$6\ ,5\dot{7}1428\dot{5}$
Take	$3\ ,6\dot{4}2857\dot{1}$	$3\ ,6\dot{4}2857\dot{1}$
Rem.		$2\ ,9\dot{2}8571\dot{4}$

(2.)

	Dissimilar	made Similar.
From	$34,4791\dot{6}$	$34,4791\dot{6}\dot{6}$
Take	$17,6\dot{8}\dot{1}$	$17,6818\dot{1}\dot{8}$
		Rem. $16,7973\dot{4}\dot{8}$

(3).

	Dissimilar	made Similar.
From	$4,\dot{6}1904\dot{7}$	$4,6\dot{1}9047\dot{6}$
Take	$1,9\dot{5}\dot{4}$	$1,9\dot{5}4545\dot{4}$
		Rem. $2,6\dot{6}4502\dot{1}$

(4.)

	Dissimilar	made Similar
From	$1,2\dot{2}\dot{7}$	$1,2\dot{2}72727272\dot{7}$
Take	$0,\dot{9}512\dot{1}$	$0,9\dot{5}12195121\dot{9}$
		Rem. $0,2\dot{7}60532150\dot{7}$

(5.)

	Dissimilar	made Similar.
From	$,8\dot{8}4615\dot{3}$	$,8846\dot{1}5384\dot{6}$
Take	$,3125$	$,3125\dot{0}0000\dot{0}$
		Rem. $,5721\dot{1}5384\dot{6}$

(6.)

(6.)

Dissimilar		made Similar.
From	10,5	$10,5\dot{0}\dot{0}$
Take	3,45	$3,4\dot{5}\dot{4}$
	Rem.	$7,0\dot{4}\dot{5}$

Illustrations of the foregoing Examples.

Ex. 1. Expressed in Vulgar Fractions is $6\frac{4}{7}$ C$^{wt.}$ — $3\frac{9}{14}$ C$^{wt.}$ $= 2\frac{13}{14}$ C$^{wt.}$ $= 2,9\dot{2}8571\dot{4}$ C$^{wt.}$ Or in other words it is from 6 C$^{wt.}$ 2 Q$^{rs.}$ 8 ℔ take 3 C$^{wt.}$ 2 Q^{rs} 16 ℔ Remainder is 2 C$^{wt.}$ 3 Q$^{rs.}$ 20 ℔.

Ex. 2. Is $34\frac{23}{48} - 17\frac{15}{22} = 16\frac{421}{528} = 16,7973\dot{4}\dot{8}$ where observe that by Transformation the given Repetend of the Subtrahend became less than the Repetend of the Minuend.

Ex 3. Is $4\frac{13}{21} - 1\frac{21}{22} = 2\frac{307}{462} = 2,6\dot{6}4502\dot{1}$ and by Transformation the given Repetend of the Subtrahend became greater, than the Repetend of the Minuend: wherefore 1, was added before the Subduction began, as was above directed.

Ex. 4. Is $1\frac{5}{22} - \frac{39}{41} = \frac{249}{902} = ,2\dot{7}60532150\dot{7}$

Ex. 5. Is $\frac{23}{26} - \frac{5}{16} = \frac{169}{208} = ,5721\dot{1}5384\dot{6}$

Ex. 6.

Ex. 6. Is $10\frac{1}{2} - 3\frac{5}{11} = 7\frac{1}{22} = 7{,}0\dot{4}\dot{5}$

N. B. This last Example I took from Dr. *Wallis's* History before named, (*Chap.* 8.) who there exhibits it, among other Examples, as impossible to give it its true Difference in a Decimal Way, mathematically exact. And the Dr. directs the Practitioner to give its Difference pretty near the Truth, *viz.* 7,04545 -|- by Approximation: whereas, by the Method above, my Answer is expressed in a Decimal Way, mathematically exact. For since his time there is so considerable an Improvement made in the Management of Decimal Fractions, that many Thousands of Examples might be produced, to each of which we can now give the Answer true to a mathematical Exactness, with very little trouble. And with much Labour it is possible to find out and express the true Answer in a Decimal Way, to any Fractions whatsoever (not Surds) most accurately.

25. Before I proceed to the next Rule, *viz.* Multiplication, I shall first shew how to multiply any Single, or Compound Circulate by 10, or 100, or 1000, &c. And, 2dly, I shall shew a Method, how to divide a Number by any Number of 9's, after a new, easy, and compendious Manner.

First.

1st, Let the Single Circulate $,\dot{4}$ $\left(=\frac{4}{9}\right)$ be given to be multiplied by 10, or 100, or 1000, &c. And the several Products will be as follows.

$$,\dot{4} \times 10 = \dot{4}, \text{ Product}$$
$$,\dot{4} \times 100 = 4\dot{4},$$
$$,\dot{4} \times 1000 = 44\dot{4},$$
$$,\dot{4} \times 10000 = 444\dot{4},$$
$$,\dot{4} \times 100000 = 4444\dot{4},$$
$$,\dot{4} \times 1000000 = 44444\dot{4}, \text{ and so on.}$$

That $,\dot{4} \times 1000000$ is equal to $44444\dot{4},$ is manifest, For $\frac{4}{9} \times \frac{1000000}{1} = \frac{4000000}{9} = 444444,4$ *&c.*

In like manner each other Product might alſo be proved to be true.

2dly, Let the Compound Circulate $,\dot{1}219\dot{5}$ $\left(=\frac{5}{41}\right)$ be given to be multiplied by 10, or 100, or 1000, *&c.* And the ſeveral Products will be as underneath.

$$,\dot{1}219\dot{5} \times 10 = 1,\dot{2}19\dot{5}$$
$$,\dot{1}219\dot{5} \times 100 = 12,\dot{1}9\dot{5}$$
$$,\dot{1}219\dot{5} \times 1000 = 121,\dot{9}\dot{5}$$
$$,\dot{1}219\dot{5} \times 10000 = 1219,\dot{5}$$
$$,\dot{1}219\dot{5} \times 100000 = 1219\dot{5},$$
$$,\dot{1}219\dot{5} \times 1000000 = \dot{1}2195\dot{1},$$
$$,\dot{1}219\dot{5} \times 10000000 = \dot{1}21951\dot{2},$$
$$,\dot{1}219\dot{5} \times 100000000 = \dot{1}219512\dot{1}, \text{ and so on.}$$

That

That 12195 × 100000000 is equal to 12195121, is thus proved, *viz.* $\frac{5}{41} \times \frac{100000000}{1} = \frac{500000000}{41} =$ 12195121,95 *&c.*

After the ſame Manner each other Product might alſo be proved to be true.

Secondly.

26. I come now to ſhew a Compendious Method, how to divide a Number by any propoſed Number of 9's.

Example (1.)

Let 574982214888787448227897 5 be given to be divided by 99999.

RULE.

Separate the Dividend beginning at the Left-hand (as underneath) into diſtinct Columns, having as many Places of Figures in each (ſupplying its Defect with 0's at the End where there is no Repetend) as are the Number of 9's in its Diviſor, (which are 5 in the above Example:) then take the Figures in the firſt Column (57498) and place them under the Figures in the 2d Column, and their Sum (79646) place under the Figures in the 3d Column, and their Sum (168433) becauſe it conſiſts of more than 5 Places of Figures, place under the Figures in the 4th and 3d Columns, and their Sum (213255) for the ſame Reaſon as the laſt, place under the 5th and 4th Columns, (as below) and ſo continue to do, as you ſee in the following Examples, unto the laſt Column: then add up the Numbers, as they ſtand placed, by 10's, and their Sum ſhall be the Quotient required.

N. B. The Figures under the laſt Column are the Remainder of the Diviſion, or they are part of the Quotient, being

being a Decimal Circulate when the Dividend consists of Integers only, as in this Example.

Operation.

	57498	22148	88787	44822	78975
		57498	79646	68433	13255
			1	2	2
Quotient	57498	79647	68435	13257	92232

$$\text{Quotient is } 57498796476843513257\tfrac{92232}{99999}$$

$$\text{Or } 57498796476843513257,\dot{9}223\dot{2}$$

Thus I have obtained its Quotient and Remainder, or its Quotient as a Mixt Compound Circulate, by Simple Addition only, the most easy of all Operations: and to do it you see it required but 23 Figures in the whole Work; whereas by the usual and common Method of Division, besides a careful Attention, it would cost the Practitioner not less than 238 Figures to obtain the Answer.

The Reason why 2 under the 5 in the last Column is there placed, is, because the Sum of that Column is 292230, which Sum should have been added to the Figures in the next Right-hand Column, if any; but there being no necessity for more Figures, the Units, in the Place of 100's of Thousands, must be placed as above; else we should want a true Remainder, or a just Mixt Quotient.

Example (2.)

Divide 979891 by 9999

Operation.

9798	91,00
	9798
	1
Quotient	97,99 8899

Quotient is $97\frac{9988}{9999}$ Or, $97,\dot{9}98\dot{8}$

The two ,o's above are placed in the laſt Column, to ſupply the Defect. Had there been more o's placed, in order to have had more Columns, the Circulate 9988 would have been as oft repeated, therefore it would have been needleſs to have annex'd more o's.

Example (3.)

Divide 3501,23022 *&c.* where 2 would infinitely repeat, by 99.

Operation.

35	01,	23	02	22	22	22	22	22	22	22	22	22	22
	35	36	59	61	83	05	27	49	71	93	15	37	59
					1	1	1	1	1	2	2	2	3

Quot. 35, 36 $\dot{5}9$ 61 84 06 28 50 72 95 17 $\dot{3}9$ 61 84

In this Example I placed 22 at pleaſure, in order to find where about the Quotient would produce a Circulate; and I found it as above mark'd off.

The

The Reaſon why I placed the 3 in the laſt Column, was, becauſe 259+22=281 (the Sum of that Column) +22=303 would be the Sum of the next lower Column: Therefore I placed the 3 found in the Place of 100's, as above.

Example (4.)

Divide 62952937047 by 999999.

Operation.

	629529	37047,0
		629529
Quot.	62952,9	999999

which is 62953 compleat for its Quotient.

Example (5.)

Divide 123333332211 by 999.

Operation.

	123	333	332	211
		123	456	788
Quot.	123	456	788,	999

Which is 123456789 compleat for its Quotient.

For $12345678\frac{999}{999}$ is equal to it.

Example (6.)

Divide 92609907390 by 999999.

Operation.

926099	07390,0
	926099
Quot. 92609,9	999999

Which is 92610 compleat.

Example (7.)

Divide 2465529966,9 by 999999.

Operation.

246552	9966,90	000000
	246552	243242
	1	1
Quot. 2465,53	243243	243243

Which is $2465,5\dot{3}2\dot{4}$

I have wrought this Example at large, that you may readily see that the 324 in its Quotient would infinitely repeat.

N. B. What the Quotient would be, when the Dividend consists of a lesser Number, or of the same Number of Places of Figures with its Divisor. See *Art* 14, 15, 16.

In order to find out the true Product in Multiplication, we are often obliged to divide by as many 9's as the Circulate consists of Places of Figures; therefore it was necessary that the Learner should be acquainted with the easiest Manner of doing it, as above: by which he might

with

with the greatest Expedition find the Quote of any Division by 9's, if required, to an hundred Places or more of Figures, with very little Trouble.

Here follows an Illustration of the Method used in the foregoing Compendious Division, Viz.

1st, Let 58476947 be given to be multiplied by 9999 in the most Compendious Way. Vide *Art.* 10.

Operation.

$$\text{Substract}\left\{\begin{array}{r} 584769470000 \\ 58476947 \\ \hline 584710993053 \\ \hline \end{array}\right. \text{ its Product.}$$

2dly. Now the Converse of this is its Proof by Compendious Division;

As divide 584710993053 by 9999.

Operation.

Divisor 9999	5847	1099	3053	The Dividend
		5847	6946	

Quotient 5847 6946 Rem. 9999 = $58476946\frac{9999}{9999}$; which is equal to 58476947. For the Rem. $\frac{9999}{9999}$ is equal to 1 or Unity. Wherefore the exact Quotient must be $58476946 + 1 = 58476947$, as above.

From this Illustration, I am persuaded every attentive Reader will, by Inspection only, more easily perceive the Rationale for the Compendious Method of dividing by any Number of 9's, than he would by any verbal Demonstration that can be offered him.

CHAP.

CHAP. IV.

Multiplication of Circulates.

A General Rule for all Cases.

REduce the Multiplicand and Multiplier to their Equivalent Single Fractions; then proceed according to the Rule prescrib'd in Multiplication of Vulgar Fractions, and the Fraction arising will be the Product compleat in a Vulgar Fraction. And if you divide its Numerator by its Denominator until o remain, or till you discover a Circulate in its Quotient, you have then the Product sought. But if neither of these happen so soon as you could wish, you may cease, when you think you have the Quotient near enough for your purpose, and may be content to give it as the Product approximately.

Example (1.)

Multiply $45,\dot{6}$ by 33

1st, Their Equivalent Single Fractions are $\frac{411}{9}$ and $\frac{33}{1}$

And $\frac{411}{9} \times \frac{33}{1} = \frac{13563}{9}$ the Product compleat in a Vulgar Fraction; which being divided as above directed, will produce for its Quotient 1507 which is a Finite Integral Product.

Example

Example (2.)

Multiply $9,\dot{3}$ by $,45$

1st, Their E. S. F. are $\frac{84}{9}$ and $\frac{45}{100}$

And $\frac{84}{9} \times \frac{45}{100} = \frac{3780}{900}$ the Product compleat in a Vulgar Fraction; which is equal to 4,2 which is a Finite Mixt Product.

Example (3.)

Multiply $65,\dot{7}$ by $7,\dot{2}$

1st, Their E. S. F. are $\frac{592}{9}$ and $\frac{65}{9}$

And $\frac{592}{9} \times \frac{65}{9} = \frac{38480}{9 \times 9}$ the Product compleat in a Vulgar Fraction; which is equal to $475,0\dot{5}$ which is a Mixt Single Circulating Product.

Example (4.)

Multiply $4,\dot{4}2857\dot{1}$ by $15,\dot{5}$

1st, Their E. S. F. are $\frac{4428567}{999999}$ and $\frac{140}{9}$

And $\frac{4428567}{999999} \times \frac{140}{9} = \frac{619999380}{999999 \times 9}$ the Product compleat in a Vulgar Fraction; which Expression is best reduced to $68,\dot{8}$ by Cultellation or Piece-meal, *viz.* 1st, By dividing its Numerator by 9, and that Quotient by 999999 as being the 9's of its Denominator.

Operation.

Operation.

$$
\begin{array}{r|l}
9 & 619999380 \\
\hline
999999 & 688888 \mid 20 \text{ the first Quotient} \\
 & \mid 688888 \\
\hline
 & 68,8888888888 \text{ the second Quotient}
\end{array}
$$

which is $68,\dot{8}$ the Product, or $6\dot{8}$,

Example (5.)

Multiply $57945,\dot{9}4\dot{5}$ by $57,\dot{7}$

That is,

Multiply $57\dot{9}4\dot{5},$ by $5\dot{7},$

1st, Their E. S. F. are $\frac{57888000}{999}$ and $\frac{520}{9}$

And $\frac{57888000}{999} \times \frac{520}{9} = \frac{30101760000}{999 \times 9}$ the Product compleat in a Vulgar Fraction, which Expression is best reduced to $33479\dot{8}7,\dot{9}8$ by Cultellation, as the last Example.

Operation.

$$
\begin{array}{r|l}
9 & 30101760000 \\
\hline
999 & 334 \mid 464 \mid 000 \mid 0 \text{ the first Quotient} \\
 & \mid 334 \mid 798 \mid \\
\hline
 & 333\ 798\ 7,\dot{9}\dot{8} \text{ the second Quotient}
\end{array}
$$

which is $33479\dot{8}\dot{7}$, the Product.

Example

Example (6.)

Multiply $14,\dot{8}5714\dot{2}$ by $7,0\dot{7}1428\dot{5}$

1st, Their E. S. F. are $\frac{14857138}{999999}$ and $\frac{70714215}{9999990}$

And their Product is $\frac{1050610143674520}{999999 \times 9999990}$ Compleat in a Vulgar Fraction; which is reduced by Cultellation as follows:

105061	014367	4520,00
	105061	119428

105061	119,428	571428	571428	571428	571428	571428
	105061	224489	795917	367345	938773	510201
			1	1	2	3

Q^t. 105,061224489795918367346938775510204081632

Dividend continued	571428	571428
	081629	653057
	3	4

Quotient continued $\dot{6}53061$ 224489

Here we have the Quotient true to 54 Figures deep, done at the Expence of a very few Figures; and the Product turns out a mixt Compound Circulate, as mark'd above. But we may be content to take the Product thus; 105,0612244897 which wants not the $\frac{1}{10000000000}$ Part of an Unit to be exact.

Example (7.)

Multiply ,$\dot{3}5\dot{1}$ by ,$\dot{2}\dot{7}$

1st, Their E. S. F. are $\frac{351}{999}$ and $\frac{27}{99}$

And $\frac{351}{999} \times \frac{27}{99} = \frac{9477}{999 \times 99}$ their Product compleat in a Vulgar Fraction, which Expression is best reduced by Cultellation as follows.

99	94	77,	00
		94	71
		1	1

95,$\dot{7}\dot{2}$ 72 the first Quote.

999	95,7	272	727	272
		957	229	956
		1	1	2

,$\dot{0}$958 $\dot{2}$30 958 230 the second Quote.

which is ,$\dot{0}9582\dot{3}$ their true Product.

Now seeing that the dividing by any Number of 9's, as hath been taught, (in *Art.* 26.) is much easier and more readily done, than any other Division whatever, except the dividing by 1, or 10, or 100, *&c.* or by 2, or 20, or 200, *&c.* I say, seeing that such Divisions are now made so very easy, from thence then appears the Advantage of letting the Compleat Products in Vulgar Fractions continue as they first occur, without reducing them to lower Expressions in a Vulgar Way, because in such Cases they will always have 9 or 9's, with or without 0's, for their Denominators,

nominators, as you see in the several Vulgar Products above.

Illustrations of the foregoing Examples.

Ex. 1. Expressed in a Vulgar Way is $45\frac{2}{3} \times \frac{33}{1} = \frac{4521}{3} = 1507$ compleat.

Ex. 2. Is $9\frac{1}{3} \times \frac{9}{20} = \frac{252}{60} = 4{,}2$ compleat.

Ex. I3. s $65\frac{7}{9} \times 7\frac{2}{9} = \frac{38480}{81} = 475{,}0\dot{5}$.

Ex. 4. Is $4\frac{3}{7} \times 15\frac{5}{9} = \frac{4340}{63} = 68{,}\dot{8}$.

Ex. 5. Is $57945\frac{35}{37} \times 57\frac{7}{9} = \frac{1114880000}{333} = 33\dot{4}79\dot{8}7{,}98$.

Ex. 6. Is $14\frac{6}{7} \times 7\frac{1}{14} = \frac{10296}{98} = 105{,}0612244$ &c.

Ex. 7. Is $\frac{13}{37} \times \frac{3}{11} = \frac{39}{407} = {,}\dot{0}9582\dot{3}$.

By the General Rule for all Cases, the Products of any given Circulating Expressions are very easily and readily obtained; and that too with little or no trouble, and without any burden to the Memory, more than is necessary to find the Product of two Vulgar Fractions; especially since the Methods of finding the Equivalent Single Fraction to any Circulating Expression, together with that of dividing by any Number of 9's, (the constant Divisors in such Cases) is now made beyond Expectation easy.

 Howe-

However, as there are other Methods for finding their Products made uſe of by the learned Mr. *Cunn*, and other great Authors ſince, who have followed him therein, I ſhall in this Place the more willingly make one of their Number; and becauſe too I ſhall add ſuch Obſervations upon the Whole, that whoever ſhall look into Mr. *Cunn*'s Examples of Multiplication for the future, may from mine eaſily underſtand the Reaſons of the various Methods, which that Author was pleas'd to make uſe of to give their true Products. But in order thereunto, I am under the Neceſſity of diſtributing the Examples in this Rule into 3 Varieties, *viz.*

Variety 1.

Examples, Where the Multiplicands conſiſt of Single or Compound Circulates, either Pure or Mixt, and their Multipliers are Finite Expreſſions, either Integral, Decimal or Mixt.

CASE I.

Examples having Single Circulates.

RULE.

When you multiply a Single Circulate, carry 1 to its next Left-hand Place for every 9 found in its Product, and ſubſcribe and mark the Overplus, if any; or if none a 0, for a Single Circulate in its Product; and proceed with the other Figures in the Multiplicand, as in Common Multiplication: ſo ſhall you obtain the true Product. But when the Multiplier conſiſts of two or more ſignificant Figures, then proceed with each of them as with its firſt Figure; and before you add their particular Products together, make their ſeveral Circulates to end together, as was taught in Addition: and their Sum ſhall be the Product ſought. *N.B.* Be careful to mark off its Fractional Part, as is taught in Multiplication of Decimals.

The

The Justness of the above Rule is manifest from this well-known Truth, *viz.* That Multiplication is a manifold Addition.

Examples.

	(1.)	(2.)	(3.)	(4.)
Multiply	$,\dot{4}$	$,\dot{8}$	$4,\dot{7}$	$15,\dot{4}$
by	$,2$	$,6$	4	$,06$
Product	$,0\dot{8}$	$,5\dot{3}$	$19,\dot{1}$	$,92\dot{6}$

(5.)	(6.)	(7.)	(8.)
$4,\dot{3}$	$3888,8\dot{7}$	$\dot{4}$	$,\dot{6}$
$,9$	$,009$	2	3
$3,9\dot{0}$	$34,9999\dot{0}$	$\dot{8}$	$2,\dot{0}$

Which is 3,9 Finite. Which is 34,9999 Finite. Which is 2 Finite

(9.)	(10.)	(11.)
$45,\dot{6}$	$,458\dot{3}$	$160,\dot{5}$
33	$,0625$	$1,75$
$137\dot{0}$	$2291\dot{6}$	$802\dot{7}$
$1370\dot{0}$	$9166\dot{6}$	$1123\dot{8}8$
$1507,\dot{0}$	$275000\dot{0}$	$160\dot{5}55$
which is 1507 Finite	$,0286458\dot{3}$	$280,97\dot{2}$

Illustrations

Illustrations of the foregoing Examples.

In *Ex.* 1. It is $\frac{4}{9} \times \frac{2}{10} = \frac{\dot{8}}{90} =,08$.

Ex. 2. It is $\frac{8}{9} \times \frac{6}{10} = \frac{48}{90} =,5\dot{3}$.

Ex. 3. It is $4\frac{7}{9} \times \frac{4}{1} = \frac{172}{9} =19,\dot{1}$.

Ex. 4. It is $15\frac{4}{9} \times \frac{6}{100} = \frac{834}{900} =,92\dot{6}$.

Ex. 5. It is $4\frac{3}{9} \times \frac{9}{10} = \frac{351}{90} =3,9$ compleat.

Ex. 6. It is $3888\frac{79}{90} \times \frac{9}{1000} = \frac{3149991}{90000} = 34,9999$ compleat.

Ex. 7. It is $\frac{40}{9} \times \frac{2}{1} = \frac{80}{9} =\dot{8},88$ &c. where 8 would infinitely repeat.

Ex. 8. It is $\frac{6}{9} \times \frac{3}{1} = \frac{18}{9} =2$ compleat.

Ex. 9. It is $45\frac{1}{3} \times 33$ vide *Ex.* 1. under the General Rule.

Ex. 10. It is $\frac{11}{24} \times \frac{1}{16} = \frac{11}{384} =,028645\dot{8}3$.

Ex. 11. It is $160\frac{5}{9} \times 1\frac{3}{4} = \frac{10115}{36} =280,97\dot{2}$.

CASE

CASE II.

Examples having Compound Circulates.

RULE.

Multiply Compound Circulates as in Common Multiplication; with this Caution, *viz.* That to every Product of the first Figure, to the Right-hand of it, be careful to add as many Units, as do mentally arise to be carried from the first Figure in the Circulate to the next Place to it; and proceed with the other Figures in the Multiplicand as usual: then mark off as many Places of Figures for the Circulate in the Product, as there are Places of Figures in the given Circulate. But when the Multiplier consists of two or more significant Figures, then proceed with each of them, as with its first Figure; and before you add their particular Products together, make their several Circulates to end together, as was taught in Addition: and the Sum of the Whole shall be the Product sought, when you have mark'd off its Circulate, as before directed.

Examples:

	(1.)	(2.)	(3.)	(4.)
Multiply	$,\dot{1}\dot{8}$	$,\dot{2}\dot{7}$	$3,\dot{8}5714\dot{2}$	$1,\dot{0}243\dot{9}$
by	$,3$	9	7	$,003$
	$,0\dot{5}\dot{4}$	$2,\dot{4}\dot{5}$	$26,\dot{9}9999\dot{9}$	$,00\dot{3}9731\dot{7}$

which is 27 Finite.

(5.)

(5.)	(6.)	(7.)
$4{,}5\dot{3}65\dot{8}$	${,}\dot{8}5714\dot{2}$	$8{,}6\dot{7}\dot{5}$
${,}75$	${,}875$	$37{,}89$
$22\dot{6}829\dot{2}$	$4\dot{2}8571\dot{4}$	$7\dot{8}08\dot{1}$
$3175\dot{6}09\dot{7}$	$5\dot{9}9999\dot{9}9$	$69\dot{4}0\dot{5}4$
$3{,}\dot{4}02439\dot{0}$	$6\dot{8}57142\dot{8}5$	$60\dot{7}\dot{2}972$
which is $3{,}40\dot{2}43\dot{9}$	${,}749\dot{9}9999\dot{9}$	$26\dot{0}\dot{2}7027$
	which is ${,}75$ Finite.	$328{,}721\dot{3}\dot{5}$

Illustrations of the foregoing Examples.

In *Ex*. 1. It is $\frac{2}{11} \times \frac{3}{10} = \frac{6}{110} = {,}0\dot{5}\dot{4}$.

Ex. 2. It is $\frac{3}{11} \times \frac{9}{1} = \frac{27}{11} = 2{,}\dot{4}\dot{5}$.

Ex. 3. It is $3\frac{6}{7} \times \frac{7}{1} = \frac{189}{7} = 27$ compleat.

Ex. 4. It is $1\frac{1}{41} \times \frac{3}{1000} = \frac{126}{41000} = {,}003\dot{0}731\dot{7}$.

Ex. 5. It is $4\frac{22}{41} \times \frac{3}{4} = \frac{558}{164} = 3{,}40\dot{2}43\dot{9}$.

Ex. 6. It is $\frac{6}{7} \times \frac{7}{8} = \frac{42}{56} = {,}75$ compleat.

In

Ex. 7. It is $8\frac{25}{37}\times37\frac{89}{100}=\frac{1216269}{3700}=328,72\dot{1}3\dot{5}.$

To find the true Product of Examples of this kind, Mr. Malcolm, *in* page (483) *gives the following Rule.*

Multiply by each Figure of the Multiplier, thus: Take first the Product of that Repetend, (of the Multiplicand) and divide it by a Number consisting all of 9's, as many as the Number of Places of the Repetend; write down the Remainder in the Product, and carry the Quote to the Product of the next Place, and go on with the other Places in common Form: And observe that this Remainder is a Repetend in every partial Product, and if it has not as many Places as the Divisor, or Repetend of the Multiplicand, you must supply the Defect with 0's on the Left; and in this State set it in the Product as the Repetend. When you have thus got all the partial Products for every Figure in the Multiplier, make all the Repetends Similar, which is done by drawing them all out as far as the first; then add them, the Sum is the Product sought, in which set the Decimal Point according to the Common Rule.

From this Gentleman's Rule we may easily perceive, that in Examples which have large Multipliers, it would often prove very tedious to find the true Product; whereas by the Rules aforegiven in this Variety, the same Purpose with his is every way answered, and that too with little or no trouble: for by multiplying the Multiplicand with that Caution, which I have there directed, you mentally divide each partial Product by 9 or 9's, according as the Repetend consists of one or more Places of Figures, *&c.*

Variety 2.

Examples, Where the Multiplicands are Finite Expressions, and the Multipliers consist of Single or Compound Circulates, either Pure or Mixt.

CASE. I.

Examples having Pure Circulates.

RULE.

1st, When the Multiplier is a Pure Single, or a Pure Compound Circulate, 1°. Let the whole Operation be as in Multiplication of Common Decimals; then multiply its Product by 1 with as many 0's annexed as the Circulate consists of Places of Figures; and with alike Number of 9's divide this last Product, so shall its Quotient be the true Product required.

Or thus,

2dly, Find the Multiplier's Equivalent Single Fraction; then with its Numerator multiply the Given Multiplicand, as in Common Decimals; and that Product divide by its Denominator, and the Quotient shall be the Product required.

N. B. In every Operation both in this and the next Variety, where the Given Multiplier is made use of, there the first Direction is followed: But where a new Multiplier is used, there the second Direction takes place.

I shall explain the above Rules by the two following Examples, wrought according to both Directions.

By the first Direction. Example (1.)

Multiply ,6 by $,\dot{8}$

Operation.

$$\begin{array}{r l}
,6 & \\
,\dot{8} & \\
\hline
,48 & \text{the first or Common Product:} \\
\hline
4,8 & \text{the C. P. multiplied by 10:}
\end{array}$$

And $9 \mid 4,8$

will give $,5\dot{3}$ for its Quotient; which is the true Product required.

(2.)

Multiply ,875 by $,\dot{3}\dot{6}$

Operation.

$$\begin{array}{r l}
,875 & \\
,\dot{3}\dot{6} & \\
\hline
5250 & \\
2625 & \\
\hline
,31500 & \text{the first or Common Product:} \\
\hline
31,500 & \text{the C. P. multiplied by 100:}
\end{array}$$

And $99 \mid 31,50$
31

will give ,3181 for its Quotient, which is its true Product required.

 The

The same Examples by the second Direction.

Multiply ,6 by $\dot{8}$ Note ,$\dot{8}$ is equal to $\frac{8}{9}$

Therefore 8 is its New Multiplier.

$$9 \,|\, \underline{4,8}$$

,5$\dot{3}$ its true Product.

Multiply ,875 by ,$\dot{3}\dot{6}$ Note ,$\dot{3}\dot{6} = \frac{36}{99}$

Therefore 36 is its New Multiplier.

	5250		
	2625		
99	31,	50	00
		31	81

,3$\dot{1}$ $\dot{8}$1 $\dot{8}$1 its true Product, which is ,3$\dot{1}\dot{8}$.

Illustrations.

Now *Ex.* 1. Is $\frac{6}{10} \times \frac{8}{9} = \frac{48}{90}$, its Product compleat in a Vulgar Fraction; whose Numerator and Denominator being divided by 10, the Expression will become $\frac{4,8}{9}$ whose Quotient is ,5$\dot{3}$, as in the Operation.

And *Ex.* 2. Is $\frac{875}{1000} \times \frac{36}{99} = \frac{31500}{99000}$ its Product compleat in a Vulgar Fraction; whose Numerator and Denominator being divided by 100, the Expression will become $\frac{315}{990} = \frac{31,5}{99}$ whose Quotient is ,3$\dot{1}\dot{8}$, as in the Operation.

Compare

Compare these and the following Illustrations with their several Examples, and you will easily see the Reason of my penning the Rules in the manner I have done under *Case* 1. in this Variety.

More Examples.

(3.)

Multiply ,015625

by $,\dot{8}$

9 | ,125000 Common Product multiplied by 10.

$,013\dot{8}$ True Product.

(4.)

M. 7,875

by $,\dot{3}$

9 | 23,625 C. P. × 10

2,625 True Product Finite.

(5.)

M. 2,54

by $,0\dot{3}$

9 | ,762 C. P. × 10

$,084\dot{6}$ True Product.

(6.)

M. 540

by $,\dot{7}$

9 | 3780 C. P. × 10

420 True Product Finite.

(7.)

M. 636

by $\dot{3}$

9 | 19080 C. P. × 10

2120 True Product Finite.

Illustrations

Illustrations of the foregoing Examples.

Ex 3. Is $\frac{1}{64} \times \frac{8}{9} = \frac{8}{576} = ,013\dot{8}$.

E. 4. Is $7\frac{7}{8} \times \frac{3}{9} = \frac{189}{72} = 2,625$ compleat.

Ex. 5. Is $\frac{254}{100} \times \frac{3}{90} = \frac{762}{9000} = ,085\dot{6}$.

Ex. 6. Is $\frac{540}{1} \times \frac{7}{9} = \frac{3780}{9} = 420$ compleat.

Ex. 7. Is $\frac{636}{1} \times \frac{30}{9} = \frac{19080}{9} = 2120$ compleat.

Obſerve when $,\dot{3}$ is the Multiplier, its true Product will be the $\frac{1}{3}$ of its Multiplicand, whether this be a Finite or a Circulating Expreſſion: For $,\dot{3} = \frac{3}{9} = \frac{1}{3}$. But when the Multiplier is either $,0\dot{3}$ or $,00\dot{3}$ or $,000\dot{3}$ *&c.* then the $\frac{1}{30}$ or $\frac{1}{300}$ or $\frac{1}{3000}$ *&c.* of the Multiplicand will be its true Product: And when the Multiplier is either $\dot{3}$, or $\dot{3}\dot{3}$, or $\dot{3}33$ *&c.* then the $\frac{1}{3}$ of 10 times, or 100 times, or 1000 times, *&c.* of the Multiplicand will be its true Product.

(8.)

(8.)

M. 1422
by $,\dot{6}7\dot{5}$

7110
9954
8532

999 | 959 | 850 C. P. × 1000
959
1

960, $\dot{8}1\dot{0}$ True Product.

(9.)

M. 500,875
by $,\dot{2}926\dot{8}$

4007000
3005250
1001750
907875
1001750

99999 | 11059 | 609,50 | 0 C. P. × 100000
11059

110,59 $\dot{7}200\dot{9}$ True Product.

Which is 110,597200

(10.)

(10.)

M. 19448100

by ,380952

38896200
97240500
175032900
1555848000
58344300

999999 | 740879 | 259120 | 0 C. P. × 1000000
740879

7408799,99999 True Product.

which is 7408800 Finite.

Illustrations of the foregoing Examples.

Ex. 8. Is $\frac{1422}{1} \times \frac{675}{999} = \frac{959850}{999} = 960,810.$

Ex. 9. Is $\frac{500875}{1000} \times \frac{29268}{99999} = \frac{11059609500}{99999000} =$ 110,597200.

Ex. 10. Is $\frac{19448100}{1} \times \frac{380952}{999999} = \frac{7408792591200}{999999} =$ 7408800 compleat.

From the great Number of Illustrations upon the Principles of Vulgar Fractions, already exhibited, I am persuaded that even no common Reader, whoever, can now stand in any farther need of my Assistance for more, but what he may very readily supply himself. And I wish I might escape Censure for having given so many. Wherefore I shall omit the exhibiting any more, and leave it to the Practitioner to apply them to such of the following Examples, as he himself shall think may require it.

CASE

CASE II.

Examples having Mixt Circulates.

RULE.

When the Multiplier is a Mixt Circulate, having in its Repetend but 1 or 2 Places of Figures, and its Finite Part consists of more Places, then 1st, Find the true Product of the given Circulate, as last directed; and 2dly, Find the Product of the remaining Figures; which add to the true Product of the given Circulate: and the last Result shall be the Product sought.

Examples.

(1.)

M. 475,75
by $14,2\dot{6}$

9 | 285,450 = C. P. of $,0\dot{6} \times 10$:

$31,71\dot{6}$ = True P. of $,0\dot{6}$:
95150 = P. of ,2:
190300 = P. of 4:
47575 = P. of 10:

$6787,36\dot{6}$

which is $6787,3\dot{6}$

(2.)

M. 675,054
by 61,03

22,50180 = True P. of ,03:
6750540 = P. of 1,0:
4050324 = P. of 60:

41200,79580

which is 41200,7958 Finite.

(3.)

M. 487,65
by 5,06

9 | 292,590 = C. P. of ,06 × 10

32,510 = True P. of ,06:
2438250 = P. of 5,0:

2470,760

which is 2470,76 Finite.

(4.)

M. 405,81
by 70,45

202905
162324

99 | 18 | 26 18 | 1,4 4 4 | 5 58 1 = C. P. of ,45 × 100

18 4,4 59 0 = True P. of ,45:
28 40 67 0 = P. of 70:

28 59 1 ,15 90 True Product.

Observe

Obſerve that in ſuch Operations as thoſe it is very neceſſary to mark off the true Product of the given Circulate with its Decimal Diſtinction, as above; that being a certain Guide where to place the next Product, for want of which an Error might eaſily ariſe.

The two following Examples are wrought according to Direction the 2d, under this Variety.

(5.)

M. ,28125 Note the Expreſſion $4,\dot{3}\dot{6} = \frac{432}{99}$

by $4,\dot{3}\dot{6}$

Subſt. 4 the Finite Part

will leave 432 the New Multiplier of the given Circulate's E. S. F.

56250
84375
112500

99 | 12 | 1,5 | 00 | 00 Here the Fractional Part is mark'd
| 1 2 | 27 | off from the Multiplicand, and its New Multiplier.

$1,2\,\dot{2}\,\dot{7}\,27$ True Product

which is $1,2\dot{2}\dot{7}$.

For as much as the Expreſſion $4,\dot{3}\dot{6}$ is equal to $\frac{432}{99}$, it is manifeſt that the Operation in all ſuch Caſes muſt be as above.

(2.)

M. 81000 Note the Expression $24,9\dot{2}\dot{5} = \frac{24901}{999}$

by $24,9\dot{2}\dot{5}$

Subst. 24

will leave 24901 the New Multiplier, &c.

81000
7290000
324000
162000

999 | 201 | 698 100 | 0, First Product :
| 201 899 | 9

201 899 9,99 9 True Product :

which is 2019000 Finite.

I must here observe to my Reader, That most commonly every Example, that may happen in Compound Circulates under this 2d Variety, is more easily and quicker solved by changing the Multiplicand for the Multiplier, & *e contra* by proceeding with the Operation as directed under *Case* 2. *Variety* 1. As for Instance; let us take the last Example, and we shall see the Advantage in so doing.

M. $24,9\dot{2}\dot{5}$
by 81 000

00
00
00 I place the o's thus, to prevent the Learner's making any Mistake in marking off the Fractional Part.
00

24925925
1994074074

2018999,999

which is 2019000 compleat.

Thus;

Thus you may (by changing the Multiplicand for the Multiplier, where Compound Circulates are given) find the true Product with the least Trouble. But I was obliged to make this 2d Variety, in order to explain some of the Operations made use of by Mr. *Cunn*, and other Authors since him.

Variety 3.

Examples, Where both Multiplicand and Multiplier consist of Circulates, either Single or Compound, Pure or Mixt.

R U L E.

1st, When the Given Multiplier is a Pure Circulate, then multiply as was taught in *Variety* 1, and divide its Product by as many 9's, as the Given Multiplier hath Places of Figures; the Quotient shall be the true Product required.

2dly, But when the Given Multiplier is a Mixt Circulate, first find its Equivalent Single Fraction, and then with its Numerator, multiply as above directed; and by its Denominator, divide that Product; the Quotient shall be the true Product required.

Examples.

(1.)

M. $11,52\dot{3}$

by $,0\dot{6}$

$9 \mid 6,914\dot{0} =$ C. P. × 10.

$,768\dot{2}$ True Product.

(2.)

M. $60,0\dot{6}$

by $,\dot{6}$

$9 \mid 360,4\dot{0} =$ C.P. × 10.

$40,0\dot{4}$.

(3.)

(3.)

M. $18,58\dot{4}$

by $,\dot{4}$

$9 \mid 74,33\dot{7}$ = C. P. × 10:

$8,2\dot{5}97530864\dot{1}$

(4.)

M. $25,3\dot{4}$

by $,0\dot{5}$

$9 \mid 12,67\dot{2}$ = C. P. × 10:

$1,40\dot{8}0246913\dot{5}$

Obſerve, When both Factors conſiſt of Circulates, it is very uncertain what kind of Circulate will ariſe in the true Product, becauſe of the Figure or Figures which repeat in the Common Product, before the dividing by 9 or 9's; therefore we muſt very often in ſuch Caſes be content with the Product approaching as near the Truth, as Neceſſity may require.

(5.)

M. $57,65\dot{6}$

by $,\dot{3}$

$19,21\dot{8}$ True Product.

(6.)

M. $57,65\dot{6}$

by 3

$192,1\dot{8}$ True Product.

(7.)

M. $5,\dot{3}$ by $\dot{6}$ Note $\dot{6}, = \frac{60}{9}$

Therefore 60 is its New Multiplier.

$9 \mid 32,00$ First Product:

$3\dot{5},$ True Product.

(8.)

(8.)

M. 54321, by 6666. Note $6666, = \frac{60000}{9}$

Therefore 60000 is its New Multiplier.

9 | 3259260000 First Product:

362140000 True Product Finite.

This *Example* is of the 2d Variety.

Compare the two preceding Examples with Mr. *Hatton's* Operations in his *Mathematical Manual*, page 187. and 189, being the 11th and 12th Propositions of his Mysterious Curiosities in Numbers, or Numerical Novelties.

(9.)

M. ,141872

by 4444,

9 | 5674,90 C. P. × 10000

630,54 True Product.

(10.)

M. 134,75

by 6,4

9 | 539,02 C. P. of ,4 × 10:

59,8913580246 = True P. of ,4

8085333333333 = P. of 6,

868,4246913580 True Product.

(11.)

(11.)

M. 351 See *Ex.* 7. under the General Rule.

by ,27

2459

7027

99	9,4	86	48	64	C. P. × 100:
		94	80	28	
		1	2	3	

,095 82 30 95 True Product ,095823.

(12.)

M. 57945,945 by 57,7

That is,

M. 57945, Note the Expression $57, = \frac{520}{9}$

by 57,

Therefore 520 is the New Multiplier

000

1158918

28972972

9 | 30131891 First Product:

3347987,9 True Product:

which is 3347987. That

That nothing might be wanting perfectly to inform the Practitioner, the next two Examples ſhall conſiſt of low Decimal Expreſſions; that when any ſuch ſhall occur, he may not be at a Loſs how to mark off the Decimal Diſtinction in the true or approximate Product.

(13.)

Multiply $,0\dot{2}\dot{7}$ Note $,0\dot{3} = \frac{3}{99}$

by $,\dot{0}\dot{3}$

Therefore 3 is the New Multiplier:

$,0\dot{8}\dot{1}$ Firſt Product:

99	,08	10	81	08	10	81	08
		08	18	99	07	17	98
				1	1	1	

$,\dot{0}008\ 1\dot{9}$ 00 08 19 00 True Product:

which is $,000\dot{8}1\dot{9}$

(14.)

M. $,\dot{0}2\dot{7}$ Note the Expreſſion $,\dot{0}2\dot{7} = \frac{27}{999}$

by $,\dot{0}2\dot{7}$

Therefore 27 is the New Multiplier:

$\dot{1}8\dot{9}$

$\dot{0}54\dot{0}$

$,7\dot{2}9$ Firſt Product.

999	,729	729	729	729	729	729 &c.
		729	458	187	916	645
		1	2	2	3	

App.P. $,\dot{0}00730$ 460 189 919 649 37 &c. the whole of which I ſhall exhibit in Involution, *Chap.* 7.

In the 1st of the two Examples above I prefixed two o's, and in the latter three o's, before I set the Decimal Distinction in their true Products; it being according to the Laws of Division in Common Decimals: which I would have the Learner carefully observe, to prevent Mistakes.

(15.)

M. 14,857142
by 7,0714285
Subst. 70

Note 7,0714285 = $\frac{70714215}{9999990}$

70714215 the New Multiplier.

742857142
1485714284
29714285712
594285714284
1485714285712
10399999999999
00000000
10399999999999999

1050611194,285714 First Product.

See its true Product, *Example* 6. under the General Rule.

I shall conclude this Variety, and with it Multiplication, by exhibiting the Operations (after my manner) of the last three Examples from Mr. *Cunn*, *pag*. 82, 83. And I doubt not but that by these Examples, and those preceding, the

the meaneſt Capacity, with very little Attention, may moſt readily be able to give or aſſign a Reaſon for all the various Methods, which that Author (or any other ſince him) was pleaſed to make uſe of, to find the true Product of any Circulating Expreſſions.

M. 3,145 Note the Expreſſion $4,\dot{2}9\dot{7} = \frac{4293}{999}$

by 4,297

Subſt. 4

will leave 4293 the New Multiplier

9436

283090

629090

12581818

999	135	03,4	363	636	Firſt Product :
		13 5	169	532	
				1	

13,5 169 533 169 True Product.

M. 2,172
by 111,98
Subst. 11

Note the Expression 111,98 = $\frac{111870}{999}$

Therefore 111870 is the New Multiplier, because the Circulation begins in the Place of Units.

00
152090
1738181
2172727
21727272
217272727

I place the first 0's to prevent the Learner's making any mistake in marking off the Fractional Part.

999	243	062,	999	First Product:
		243	305	
			2	

243, 306 306 True Product.

N. B. Because in the common Product the Circulate is 99, therefore Mr. *Cunn* judiciously added 1 to the next Place; thereby making the 2 become a 3: but I have wrought it at large, to let the Reader see that either way the true Product would turn out the same.

Multiply 21485,314 Note 481,7652 = $\frac{481764800}{999999}$

by 481,7652

Subst. 4

will leave 481764800 the New Multiplier, because the Circulation begins in the Place of Tens.

000

0000

171882514 51

85941257257

128911885885 8

15039720020020

21485314314314

171882514514514 5

85941257257257 25

103508681535 72,772 First Product, which for conveniently dividing by 999999, I take down as underneath.

103508	681535	72,7727	727727	727727	727 &c.
	103508	785043	512770	240497	968 &c.
		1	2	2	

103508 78,5044 512772 240499 968227 69.

If it be desired to find the Value of that Fraction, which is left out, observe, that by omitting as many of the last Places of Figures, as the Repetend of the Multiplier did consist

consist of, *viz.* 6 Places, then the Approximate Product will be 10350878,504451277224099996; which wants of the true Product $\frac{822769\frac{772}{999}}{999999}$ of an Unit in the last Place, the Place of 6 after the 3 Nines.

Now the Expression $\frac{822769\frac{772}{999}}{999999}$ is equal to $\frac{821947003}{998999001}$, and the 6 is in the 18th Place below Unity, therefore the approximate Product is defective of being the true Product in the Quantity $\frac{821947003}{998999001}$ of One, one trillionth Part of an Unit, the Value of the Fraction left out.

Whoever will take the Pains to compare these last three Operations of mine with those of Mr. *Cunn*, will perceive that their first Products widely differ in their Fractional Parts.

The Reason, as I take it, (for he assigns none) must be as follows: He must have considered the several New Multipliers as Decimal Expressions, *viz.* his first he must have considered as $\frac{4,293}{,999}$; his second as $\frac{111,870}{,999}$; and his last as $\frac{481,764800}{,999999}$.

And his having thus considered them, is the Reason why he omitted the o's in the last two Multipliers. For o's at the End of Decimals neither increase nor diminish the Product.

Also

Alſo from the Expreſſions being conſidered as above, ariſes the Reaſon why his Quotient contains the like Number of Integral Places of Figures with thoſe in his firſt Product. For any Integral Numbers (not all 9's) being to be divided by as many 9's Decimally expreſſed, as the Given Number contains Places of Figures, will give in its Quotient the like Number of Integral Places.

CHAP.

CHAP. V.

Division of Circulates.

A General Rule for all Cases.

REduce the Divisor and Dividend to their Equivalent Single Fractions, then proceed according to the Rule prescribed in Division of Vulgar Fractions; and the Fraction arising will be the Quotient compleat in a Vulgar Fraction. And if you divide its Numerator by its Denominator, until o remain, or 'till you discover a Circulate in its Quotient, you have then obtained the true Quotient sought. But if neither of those happen so soon as you could wish, you may cease, when you think you have the Quotient near enough for your purpose, and may be content to give it as the Quotient approximately.

Example 1.

Divide $19,\dot{1}$ by 4.

1st, Their Equivalent Vulgar Fractions are $\frac{172}{9}$ and $\frac{4}{1}$;

And $\frac{172}{9} \div \frac{4}{1} = \frac{172}{36}$ the Quotient compleat in a Vulgar Fraction; which being divided, as above directed, will produce for its Quotient $4,\dot{7}$.

Example 2.

Divide $115,\dot{4}$ by ,6.

1st, Their E. S. F. are $\frac{1039}{9}$ and $\frac{6}{10}$;

And $\frac{1039}{9} \div \frac{6}{10} = \frac{10390}{54}$ the Quotient compleat in a Vulgar Fraction; which is equal to $192,\dot{4}0\dot{7}$.

Example

Example 3.

Divide 2470,76 by $5{,}0\dot{6}$.

1st, Their E. S. F. are $\frac{247076}{100}$ and $\frac{456}{90}$;

And $\frac{247076}{100} \div \frac{456}{90} = \frac{22236840}{45600}$ the Quotient compleat in a Vulgar Fraction; which is equal to 487,65 Finite.

Example 4.

Divide 579,6 by $,\dot{2}4\dot{3}$.

1st, Their E. S. F. are $\frac{5796}{10}$ and $\frac{243}{999}$;

And $\frac{5796}{10} \div \frac{243}{999} = \frac{5790204}{2430}$ the Quotient compleat in a Vulgar Fraction; which is equal to 2382,8 Finite.

The Proof.

Multiply $,\dot{2}4\dot{3}$ the Divisor
by 2382,8 the Quotient.

$$
\begin{array}{r}
1\dot{9}4\dot{5} \\
\dot{4}8\dot{6}4 \\
1\dot{9}4\dot{5}94 \\
\dot{7}2\dot{9}729 \\
\dot{4}8\dot{6}4864 \\
\hline
579{,}5\dot{9}9\dot{9} \\
\hline
\end{array}
$$

which is 579,6 the Dividend as above.

Example 5.

Divide $,\dot{1}6\dot{7}$ by $,7\dot{5}$.

1st, Their E. S. F. are $\frac{167}{999}$ and $\frac{68}{90}$;

And $\frac{167}{999} \div \frac{68}{90} = \frac{15030}{67932}$ their Quotient compleat in a Vulgar Fraction; which is equal to $,22\dot{1}250662427133015368309485956544780074191\dot{8}388977$.

Example 6.

Divide $10124,977\dot{1}\dot{7}$ by $\dot{2}3,41\dot{4}$.

1st, Their E. S. F. are $\frac{1002372740}{99000}$ and $\frac{2341400}{99999}$, whose Quotient is $\frac{100236271627260}{231798600000}$ compleat in a Vulgar Fraction; which is equal to $432,4282874325384\frac{443376}{2317986}$. I set the Answer thus, that he, whose Curiosity prompts him, may proceed to find its Repetend.

As in Multiplication, so here in Division, we must very often be content to give the Quotient approximately.

After the manner above might all the Examples in this Rule be readily solved. However, to comply with the Custom of other Authors, I shall here exhibit other Methods, the Foundation of whose Operations chiefly depend on the foregoing general Method. And in order for the Learner's more ready Apprehension, I shall distribute all the Examples, which can occur in this Rule, into three Varieties, *viz.*

Variety

Variety (1.)

When the Divifor is a Finite Expreffion, and its Dividend confifts of a Circulate either Single or Compound, Pure or Mixt, obferve the following Rule:

Divide as in Common Decimals; but be careful in the Operation to apply the given Circulate in the Dividend fo oft, 'till the Quotient turns out a Circulate: or if that does not happen fo foon as you could wifh, you may be content to give your Quotient approximately. For indeed in many Examples that may occur in Practice, except where the Divifor is a fingle Digit, and its Dividend confifts of a fingle Circulate, on fome chofen Product with one of its Factors, the Operation will frequently prove very tedious, if you are determined to find its circulating Quotient.

Examples.

(1.)

$7 \mid 756{,}\dot{7}$

Quote $118{,}\dot{1}$

(2.)

$,06 \mid ,92\dot{6}$

Quote $15{,}\dot{4}$

(3.)

$5 \mid ,0\dot{4}5$

Quote $,0\dot{0}\dot{9}$

(4.)

$,6 \mid 7175{,}\dot{3}$

Quote $1195\dot{8}{,}$

(5.)

(5.)

Divide $5\dot{8}\dot{1}$ by 8.

8 | 58$\dot{1}$,81

Quote 72,72

which is $\dot{7}\dot{2}$,

(6.)

Divide $585,\dot{4}\dot{2}$ by ,7.

,7 | 585,4$\dot{2}$4242

Quote $8\dot{3}6,3203\dot{4}$

(7.)

Divide $4,\dot{8}57142$ by 7.

7 | 4,857142857142857142857142857142857142857142

Q^te ,$\dot{6}$93877551020408163265306122448979591836734

I took down the three last Examples for the Conveniency of applying the given Circulate; which I repeated until the Quotient turned out a Circulate.

(8.)

(8.)

Divide $96,\dot{3}7\dot{8}$ by 58.

$58 \mid 96,\dot{3}7\dot{8}$ | 1,661696178

58
383
348
357
348
98
58
403
384
557
522
358
348
103
58
457
406
518
464
54

Answer, 1,661696178 the Quotient approximately.

Thus, I think, I have exhibited Examples enough under this Variety. If any one have an Inclination to practise with such as will turn out Circulates, I refer him to the Examples in Multiplication, where he will meet with many whose Products being a Circulate or Finite Expression, if divided

ivided by one of its Factors, will give the other for its Quotient. For Division is the best Proof of Multiplication, as this is of that.

Variety (2.)

When the Divisor consists of a Circulate, either Single or Compound, Pure or Mixt, and its Dividend of a Finite Expression, observe the following Directions.

RULE. 1.

Find the Divisor's Equivalent Single Fraction; then with its Denominator, (which in this Variety and in the next will always be 9 or 9's, with or without 0's) considered as an Integral Number, multiply the Dividend; and this Product divide by the Divisor's Numerator, considered as an Integral Number, the Quotient arising shall be that sought.

CASE I.

Of Pure Circulates, either Single or Compound.

Examples.

(1.)

Divide 5664 by $,\dot{8}$.

$,\dot{8}$ | 5664 Here $\frac{8}{9}$ is the Divisor's E. S. F.
9

8 | 50976 New Dividend.

Quote 6372 Finite.

(2.)

Divide 746,3 by $,\dot{5}$.

$,\dot{5}$ | 746,3 Here $\frac{5}{9}$ is the Divisor's E. S. F.
9

5 | 6716,7 New Dividend.

Q^{te} 1343,34 Finite.

I here beg leave to illustrate those Examples, and with them the Rule above, by working both Examples after the manner of Vulgar Fractions.

(1.)

(1.)

Divide 5664 by $,\dot{8}$.

1st, Their E. S. F. are $\frac{5664}{1}$ and $\frac{8}{9}$

Divisor $\frac{8}{9}$ $\qquad\qquad$ $\frac{5664}{1}$ Dvidend.

$$\begin{array}{rcr} 5664 & & 8 \\ \hline 50976 \text{ Num}^{r.} & & 8 \text{ Denom}^{r.} \end{array}$$

its Quote is $\frac{50976}{8} = 6372$ compleat.

(2.)

Divide 746,3 by $,\dot{5}$.

1st, Their E. S. F. are $\frac{7463}{10}$ and $\frac{5}{9}$

Divisor $\frac{5}{9}$ $\qquad\qquad$ $\frac{7463}{10}$ Dividend.

$$\begin{array}{rcr} 7463 & & 5 \\ \hline 67167 \text{ Num}^{r.} & & 50 \text{ Denom}^{r.} \end{array}$$

its Quote is $\frac{67167}{50} = 1343,34$ compleat.

Note, The Expression $\frac{67167}{50}$ is the same with $\frac{6716,7}{5}$.

Whoever, with a little Attention, will look into these last Operations, will soon discover the Reason of my penning the Rule as above.

For Examples in this Variety, where the Divisor is only a single Repetend, Mr. *Martin* to find the true Quotient directs thus:

RULE.

RULE.

Multiply the Dividend by 9, cutting off one more Right-hand Figure in the Product, which is now your new Dividend; then divide as usual, and the Quotient will be just.

Let us resume my last Example, and work it by his Direction.

Divide 746,3 by ,$\dot{5}$.

The Dividend = 746,3
Multiply by 9

Divisor = ,5 | 671,67 = New Dividend

1343,34 = True Quotient.

Which is the same Quote as mine.

Mr. Pardon's *Rule is,*

If your given Divisor be a Single Repetend, and your Dividend a terminate Number, multiply the Dividend by ,9, and divide that Product by the given Divisor. As,

Divide 746,3 by ,$\dot{5}$.

The Dividend = 746,3
Multiply by ,9

Divisor =,5 | 671,67 = New Dividend.

1343,34 = True Quotient.

Here, it is manifest that by multiplying by ,9, according to Mr. *Pardon*, instead of 9, the New Dividend in this Operation becomes the same with Mr. *Martin*'s; and my dividing by 5, and not ,5, produces the same true Product with either. Hence though they may seem to some three different Rules, and by that means puzzle the young Tyro to reconcile them, yet their Effects you see are one and the same, and all have the same Foundation in Nature.

My

My Method being built partly upon the Principles of Vulgar Fractions, and partly on that of Decimals; and their Method wholly on that of Decimals.

But observe, that in either of the three Methods aforegoing, when the New Dividend is found, the Infinite Divisor in the Operation then becomes a Finite Expression; else the Laws of multiplying and substracting of Circulates ought to be observed, as underneath, where the same Example is wrought at large.

,5 | 746,30 *in Infinitum.*

55555 *in Infinitum.*

1343,34 Quotient

19074 &c.

16666

2407

2222

185

166

18

16

22

22

0

By this Operation you may perceive that there is an Infinite Product for every Infinite Remainder, and so continues to be repeated until the Infinity vanishes into o, *the Universal Symbole or Character for Infinity.*

(3.)		(4.)	
$,\dot{7}$ | $68,743$		$,\dot{7}$ | $10,45$	
9		9	
7 | $618,687$	New Div$^{d.}$	7 | $94,05$	New Div$^{d.}$
Quote $88,38\dot{3}857142\dot{}.$		Quote $13,4\dot{3}57142\dot{8}.$	

I ſhall take the Liberty here to remind the Learner,

$$\text{That the Expreſſion}\quad \left.\begin{array}{l} ,000\dot{6} = \dfrac{6}{9000} \\ ,00\dot{6} = \dfrac{6}{900} \\ ,0\dot{6} = \dfrac{6}{90} \\ ,\dot{6} = \dfrac{6}{9} \\ \dot{6}, = \dfrac{60}{9} \\ \dot{6}6, = \dfrac{600}{9} \\ \dot{6}66, = \dfrac{6000}{9} \\ \dot{6}666, = \dfrac{60000}{9} \end{array}\right\}\ \text{Their ſeveral E. S. F.}$$

and then propoſe the following Queries, *viz.*

Let 742,85 be given to be divided by each of the Infinite Expreſſions above.

(5.)

(5.)

1st, $,\dot{0}006$ | 742,85 the Dividend
9000 the Denominator as above

6 | 6685650,00 the New Dividend

1114275, True Quotient.

(6.)

8th, $\dot{6}666$ | 742,85 the Dividend
9 the Denominator as above

60000 | 6685,65 the New Dividend

,1114275 True Quotient.

I have exhibited the Operations of the first and last Examples, leaving the intermediate ones as an Exercise for the Learner.

(7.)

Divide 6794 by $5,\dot{1}\dot{8}$.

Now $5,\dot{1}8 = \frac{5180}{999}$.

6794000
6794

5180 | 6787206 = Dividend × 999;

$1310,27\dot{1}428\dot{5}$
true Quotient.

 It

It is not neceſſary to expreſs ſuch Diviſions at large ; becauſe the Operation is the ſame, as in Diviſion of Common Decimals.

The ſame Example, wrought wholly upon the Principles of Decimals, would be as underneath.

$$\dot{5},1\dot{8} \,\Big|\, \begin{array}{r} 6794 \\ 6794 \\ \hline \end{array}$$

$$5,18 \mid 6787,206 = \text{Dividend} \times ,999\,;$$

$1310,2\dot{7}1428\dot{5}$
true Quotient as above.

And then the Rule under this 2d Variety ſhould be expreſſed thus to make it as univerſal.

RULE 2.

Find the Diviſor's Equivalent Single Fraction ; then with its Denominator, *&c.* conſidered as a Decimal Expreſſion, multiply the Dividend ; and this Product divide by the Given Diviſor's Numerator, conſidered as a Finite Mixt Expreſſion, if the Diviſor be mixt, or conſidered as a Finite Integral Expreſſion, if the Diviſor be Integral, and the Quotient ariſing ſhall be that ſought.

Example (8.) *By this laſt Rule.*

Divide $5794,875$ by $56,\dot{0}9\dot{7}$.

$$\dot{5}6,09\dot{7} \,\Big|\, \begin{array}{r} 5794,87500000 \\ 5794875 \\ \hline \end{array}$$

$$56,097 \mid 5794,81705125 = \text{Dividend} \times ,99999\,;$$

$103,29994565217$ *&c.*
An approximate Quotient.

The

The same Example by the first Rule.

Now $5\dot{6},09\dot{7} = \frac{5609700}{99999}$

$$\begin{array}{r|l} & 5794,87500000 \\ & 5794875 \\ \hline 5609700 & 5794817205,125 = \text{Dividend} \times 99999; \end{array}$$

which by Inspection only we can plainly see must produce the same Quotient as before.

Thus I have exhibited two Ways to work all Examples by, that fall under this Variety, whose Divisors are Pure Circulates, whether Single or Compound.

CASE II.

Of Mixt Circulates.

1st, When the Divisor consists of a Mixt Circulate, having Decimal Places in it, and its Dividend a Finite Expression, if you work the Examples wholly in a Decimal Way, the Rule is the same with the last.

Examples.

(1.)

Divide 2470,76 by $5,0\dot{6}$.

$$\begin{array}{lr|l} & 5,0\dot{6} & 2470,76 \\ \text{Subst.} & 50 & 247076 \\ \hline & 4,56 & 2223,684 = \text{Dividend} \times,99 \end{array}$$

487,65
True Quotient Finite.

I omit the Operation at large, for the Reason before given.

(2.)

(2.)

Divide 2019000 by 24,925.

	24,925	2019000
Subst.	24	2019000
	24,901	20079^{8}1,000 Dividend ×,999
	81000	

True Quotient Finite and Integral.

(3.)

Divide 6794,75 by 753,658.

	753,658	6794,7500000
Subst.	7	679475
	753,651	6794,6820525 = Dividend ×,99999
	9,015^{8}073306 &c.	

An approximate Quotient.

2dly, When the Divisor consists of a Mixt Circulate, having Integral Places only, and its Dividend a Finite Expression. As,

(4.)

Divide 698,4 by 6347, *Vide* the last Rule.

	6347	698,4
Subst.	6	6984
	6341,	697,7016 = Dividend ×,999
	,110030216	

an approximate Quotient.

Variety

Variety (3.)

Examples where both Divisor and Dividend consist of Circulates.

I might have included all the Examples, which can occur in this Variety, under the Directions of the 2d; but Mr. *Cunn* not having exhibited any Examples in Division, which fall under my first or second Variety, I was the more willing, for distinction sake, to call this a third Variety. And indeed it is something remarkable in that Gentleman, who in his Preface complains that the Reverend Mr. *Brown* in Division leaves the Practitioner to work without Exactness, that he himself in the same Rule should leave more than half of its necessary Directions untouched; or not so much as hinted at.

27. But previously to the Examples in this Variety, I shall here shew the Learner a compendious Method of multiplying any circulating Expression by any Number of 9's.

RULE.

As many 9's as the Multiplier consists of, so many times write down the given Circulating Figure, if Single; but if Compound, transpose them alternately as you see below; then substract (according to the Laws of Substraction before directed) the given Multiplicand from itself thus transformed: and the Difference, when mark'd off as in common Decimals, shall be the Product sought.

Examples of Mixt Single Circulates.

Let $5,\dot{7}$ be multiplied by 9 or 99 or 999 *&c.*

Operation.		*Operation.*	
	$5,\dot{7}7$		$5,\dot{7}77$
Subst.	$5\dot{7}$	Subst.	$5\dot{7}$
Product	$52,0 = 5,\dot{7} \times 9$	Product	$572,0 = 5,\dot{7} \times 99$

Operation.		*Operation.*	
	$5,\dot{7}777$		$5,\dot{7}7777$
Subst.	$5\dot{7}$	Subst.	$5\dot{7}$
P.	$5772,0 = 5,\dot{7} \times 999$	P.	$57772,0 = 5,\dot{7} \times 9999$

Operation.		*Operation.*	
	$5,\dot{7}77777$		$5,\dot{7}777777$
Subst.	$5\dot{7}$	Subst.	$5\dot{7}$
P.	$577772,0 = 5,\dot{7} \times 99999$	P.	$5777772,0 = 5,\dot{7} \times 999999$

and so on for any single Circulate whatever.

Examples

Examples of Mixt Compound Circulates.

Let $6,\dot{7}\dot{5}$ be multiplied by 9 or 99 or 999 *&c.*

Operation.

$6,\dot{7}\dot{5}7$

Subst. $6\dot{7}\dot{5}$ observing the Laws of Subtraction

Product $60,\dot{8}\dot{1} = 6,\dot{7}\dot{5}\times 9$

Operation.	*Operation.*
$6,\dot{7}\dot{5}75$	$6,\dot{7}\dot{5}757$
Subst. $6\dot{7}\dot{5}$	Subst. $6\dot{7}\dot{5}$
P. $669,\dot{0}\dot{0} = 6,\dot{7}\dot{5}\times 99$	P. $6750,\dot{8}\dot{1} = 6,\dot{7}\dot{5}\times 999$

Operation.	*Operation.*
$6,\dot{7}\dot{5}7575$	$6,\dot{7}\dot{5}75757$
Subst. $6\dot{7}\dot{5}$	Subst. $6\dot{7}\dot{5}$
P. $67569,\dot{0}\dot{0} = 6,\dot{7}\dot{5}\times 9999$	P. $675750,\dot{8}\dot{1} = 6,\dot{7}\dot{5}\times 99999$

So in like manner $47\dot{9}2,\dot{5}\times 9$ will give $4313\dot{3},3$ for its Product: And $47\dot{9}2,\dot{5}\times 99 = 4744\dot{6}6,6$. And $47\dot{9}2,\dot{5}\times 999 = 4797800$. A Finite Integral Number, *&c.*

If any Pure Single Circulate is multiplied by 9, as suppose $,00\dot{7}$, its Product will be the same Figure Finite in

the next Left-hand Place (*viz.*) ,07. If multiplied by two 9's, its Product will be the same Figure Finite twice repeated in the next Left-hand Places: And so on, according to the Number of 9's in the Multiplier.

Examples of Pure Single Circulates.

As $,00\dot{7} \times 99 = ,77$. And $,00\dot{7} \times 999 = 7,77$. And $,00\dot{7} \times 9999 = 77,77$. And $,00\dot{7} \times 99999 = 777,77$. and so on.

So likewise $\dot{6} \times 9 = 60$ *&c.*

Examples of Pure Compound Circulates.

Let $,\dot{8}7514\dot{2}$ be multiplied by 9 or 99, or 999 *&c.*

Op. by 9.	*Op.* by 99.	*Op.* by 999.
$,\dot{8}7514\dot{2}8$	$,\dot{8}7514\dot{2}87$	$,\dot{8}7514\dot{2}875$
Subst. $\dot{8}7514\dot{2}$	Subst. $\dot{8}7514\dot{2}$	Subst. $\dot{8}7514\dot{2}$
P. $7,\dot{8}7628\dot{5}$	P. $86,\dot{6}3914\dot{4}$	P. $874,\dot{2}6773\dot{2}$

Op. by 9999.	*Op.* by 99999.
$,\dot{8}7514\dot{2}8751$	$,\dot{8}7514\dot{2}87514$
Subst. $\dot{8}7514\dot{2}$	Subst. $\dot{8}7514\dot{2}$
P. $8750,\dot{5}5360\dot{8}$	P. $87513,\dot{4}1237\dot{1}$

Op.

Op. by 999999.

	,875142875142
Subst.	875142
P.	875142, Finite.

Observe, That if the several given Circulating Expressions in the preceding Examples had been all Integral Numbers, then their several Products would also have been Integrals, either Finite or Circulating, having the same Figures as you see in the several Products.

But that nothing might be wanting to compleat this Rule of compendiously multiplying any kind of circulating Expressions by any Number of 9's. Let us suppose that such as 6666, or 87444, or 579467, &c. being all Integral Numbers, were given to be multiplied by any Number of 9's; their Products (which in such Cases are all Integrals) are also obtained by observing the same Laws as are before prescribed.

As, let 6666 be given to be multiplied by 9 or 99 or 999 &c.

Op. by 9.	*Op.* by 99.	*Op.* by 999.
66666	666666	6666666
Subst. 6666	Subst. 6666	Subst. 6666
P. 60000 Finite.	P. 660000 Finite.	P. 6660000 Finite.

and so on.

And let $8\dot{7}444$ be given to be multiplied by 9 or 99 or 999 *&c.*

	Op. by 9.	*Op.* by 99.
	$8\dot{7}4444$	$8\dot{7}44444$
Subst.	$8\dot{7}444$	$8\dot{7}444$
P.	787000 Finite.	8657000 Finite, and

so on.

And let $5\dot{7}94\dot{6}7$ be given to be multiplied by 9 or 99 or 999 *&c.*

Op. by 9.

$57\dot{9}46\dot{7}9$ Note $5\dot{7}94\dot{6}7 = 57\dot{9}46\dot{7}$.

Subst. $57\dot{9}46\dot{7}$ observing the Laws of Subtraction.

P. $52\dot{1}52\dot{1}1, = 5\dot{7}94\dot{6}7 \times 9$.

	Op. by 99.		*Op.* by 999.
	$57\dot{9}46\dot{7}94$		$579\dot{4}67\dot{9}46$
Subst.	$57\dot{9}46\dot{7}$	Subst.	$57\dot{9}46\dot{7}$
P.	$573\dot{6}73\dot{2}6$,	P.	$5788\dot{8}4\dot{7}8$,

Op. by 9999.

$57\dot{9}46\dot{7}9467$

$57\dot{9}46\dot{7}$

P. 579400000 Finite, and so on.

And

And laſtly, let 10124,97717 be given to be multiplied by 99999.

Operation.

10124,9771717171

1012497717

P. 1012487592,19454

Obſerve, I have conſidered the ſeveral preceding Multipliers, as ſo many Integral Numbers; and if they had been ſo many Decimal Expreſſions, then we muſt have mark'd off as many more Places of Figures in their ſeveral Products, as the Multipliers conſiſted of Decimal Places. As for Inſtance, if the Multiplier in the laſt Example had been a Decimal Expreſſion, its Product would have been 10124,8759219454.

From the preceding Products many more Remarks might be made: but, to avoid Prolixity, I ſhall make but this one, *viz.* That where any Circulating Expreſſion is multiplied by as many 9's, as the given Circulate conſiſts of Places of Figures, or any Multiple thereof, there the Product always turns out a Finite Expreſſion.

I am perſuaded it would be an entertaining Exerciſe, as well as an Improvement, for the Learner to prove ſome of the foregoing Products, by dividing them by their Multipliers after the compendious Manner of dividing by any Number of 9's.

Here follow the Examples which fall under this Variety. And if you work them wholly in a Decimal Way, as are all the following Examples, obſerve the Directions under the laſt Rule in *Variety* 2, to which I refer the Reader.

(1.)

Divide 8,724 by ,5.

,5	8,7244
	8724

New Divisor ,5 | 7,8520 New Dividend

Quotient 15,704 Finite.

(2.)

Divide 459,68 by 7.

7	459,688
	45968

New Divisor 7 | 413,720 New Dividend

Quotient 59,10285714.

(3.)

Divide 78,048 by ,08.

	,08	78,0487
Subst.	0	7 8048

,c8 | 70,2439 New Dividend

Quotient 878,04.

(4.)

Divide 42,523809 by ,3.

,3	42,5238095
	42523809
,3	38,2714285 New Dividend
Quotient	127,571428.

Note, Where the Divisor is ,3 there three times the given Dividend will also be its true Quotient. If ,03, then thirty times, and so on. Which is the Converse of the Observation made in Multiplication, *page* (72.)

(5.)

Divide 27,65 by ,08.

,08	27,6555
	2765
,08	27,3790 New Dividend
Quotient	342,2375 Finite.

(6.)

Divide 630,54 by 4444,

	4444,4	630,545
Subst.	4444	63054
	4000,0	567,490 New Dividend

Quotient ,141872

(7.)

Divide 7623,37 by 666666666,

	666666666,6	7623,373
Subst.	666666666	762337
	600000000,0	6861,036 New Dividend

Quotient ,00001143506

Thus you have a Method how to divide by any single Digit infinitely repeated, whether it begins any where in the Integral or Decimal Places.

(8.)

(8.)

Divide 243,306 by 111,98.

```
111,98 | 243,306306  } The last three Figures
    11 |    243306  }   in both might have
                    }   been omitted.
-------------------
111,87 | 243,063000 New Dividend
-------  22374
         -----
2,172     19323
Quotient. 11187
          -----
          81360 ]
          78309 |
          ----- |
          30510 } in Infinitum.
          22374 |
          ----- |
           8136 ]
```

(9.)

Divide ,095823 by ,351.

```
,351 | ,095823095
     |    095823
-----------------
,351 | ,095727272 New Dividend
---------  702
           ---
,27       2552 }
Quotient  2457 } in Infinitum.
          ---- }
           095 }
```

(10.)

Divide ,167 by ,75.

,75	,1671
7	167
,68	,1504 New Dividend
,22125 &c.	136
	144
	136
	85
	68
	170
	136
	344
	340
	4 &c.

Quotient ,22125 &c. *Vide* the true Quotient, under *Example* 5. in the General Rule; which also might here be found by carrying on of the Division, and applying the Figures of the New Circulate 504 alternately.

(11.)

Divide 120,54 by 46,21.

	46,21	120,54545
Subst.	4	12054
	46,17	120,42490 New Dividend

Quotient 2,60829346092 &c. approximately, found by carrying on the Division, and applying of the Figures of the New Circulate 90 alternately.

(12.)

(12.)

Divide ,5952380 by ,4681.

	,4681	,595238095	
Subst.	46	5952380	
	,4635	,589285714	New Dividend

Quotient 1,27138233934 *&c.* approximately.

(13.)

Divide 8,63 by ,07317.

,07317	8,6363636	
	863	
,07317	8,6362772	New Dividend
	7 317	
118,03 Quotient		
	13192	
	7317	
	58757	
	58536	
	22172	
	21951	*in Infinitum.*
	221	

Examples of Integrals.

(14.)

Divide 3347987, by 57945,

	57945	3347987,987	
Subst.	57	3347 987	
	57888	3344640,	New Dividend, the Multiplier being ,999.
		289440	
	57,	450240	*in Infinitum.*
	Quotient	405216	
		45024	

Obſerve, When the given Dividend conſiſts of the ſame Number of Places of Figures in its Circulate with thoſe of its Diviſor, or does conſiſt of ſome aliquot Part thereof, then its New Dividend will turn out a Finite Expreſſion.

I chuſe to exhibit the following Example, becauſe it is a Proof to the preceding one.

(15.)

Divide $33\dot{4}79\dot{8}7$ by $5\dot{7}$.

$$
\begin{array}{r|l}
5\dot{7} & 33\dot{4}79\dot{8}7,9 \\
\text{Subst.}\quad 5 & 3347\dot{9}8\,\dot{7} \\
\hline
52 & 3013\dot{1}8\dot{9},1 \quad \text{New Dividend}
\end{array}
$$

$$
\begin{array}{ll}
 & 260 \\
\hline
57\dot{9}4\dot{5}, & 413 \\
\text{Quotient} & 364 \\
\hline
 & 491 \\
 & 468 \\
\hline
 & 238 \\
 & 208 \quad \textit{in Infinitum.} \\
\hline
 & 309 \\
 & 260 \\
\hline
 & 49
\end{array}
$$

Before I leave this Variety, I cannot help taking Notice of two very particular things, *viz.* First, that I never met with an Example (in all the Authors I have seen on this Subject) where the Circulate in its Dividend by Transformation required in Substracting it to carry one to its Right-hand Column, as in my preceding Examples *viz.* the 4th, 6th, 11th, 12th, 13th, and 15th, which I contrived on purpose.

And 2dly, I do not remember that I ever met with an Example, where the Circulate in its Dividend consisted of fewer Places of Figures than that of its Divisor, as in my Examples 11. and 13, except One, and that, for want of a due Attention in the Proposer, is wrought falsely.

CHAP.

CHAP. VI.

Reduction of Circulates.

CASE I.

TO reduce Money, Weights, Time, or Measures, *&c.* to their Equivalent Decimal Expressions, or near it, I know not a readier Method than the following Rule.

First, Reduce (by Division) the Number of the lowest Species, given in the Example, to the Decimal of that next above it, whether there be any Number of that Species in the Example or not; to this Quotient add the Number of that Species in the Example, if there be any. And 2dly, This last Sum reduce to the next higher Species, adding to this Quotient found, the Number of that Species given in the Example, if there be any; and so proceed, until you arrive to the Decimal of the Integral sought.

Example (1.)

S. D. Qrs.
Reduce 13 : 11 : 3 to the Decimal of a *L.* Sterling.

4 | 3, Q^rs.

12 | 11,75 *D.* with 11 *D.* added to ,75 *D.* the 1st Q^te.

2 | 0 | 13,9791$\dot{6}$ *S.* with 13 *S.* added to ,9791$\dot{6}$ *S.* the 2d Q^te.

Answ. ,698958$\dot{3}$ of a *L.* Sterling.

To contract the Operation.

Observe, When any Divisor greater than 12 is composed of two Digits, either with or without 0's; or of 12's, and some other Digits, either with or without 0's; then divide by the Divisor's composed Numbers alternately: and the last Result shall be the Quote sought.

Example (2.)

Ozs. Dwt. Grs.
Reduce 10 : 13 : 14 to the Decimal of a ℔ Troy.

Instead of 24 { 6 | 14, Grains
4 | 2,3

20 | 13,583 Dwt.

12 | 10,67916 Ozs.

Answer ,8899305 of a ℔ Troy.

Example (3.)

Cwt. Qrs. ℔
Reduce 14 : 1 : 1 to the Decimal of a Tun.

28 { 4 | 1, ℔
7 | ,25

4 | 1,03571428 Qrs. Cwt.

20 | 14,2589285714 Cwt.

Answer ,712946428571 of a Tun.

Example

Example (4.)

Reduce 39 Galls. : 7 Pints. to the Decimal of an Hogshead of 63 Gallons.

8 | 7, Pints

63 { 9 | 39,875 Gallons

7 | 4,4305

Answer ,632936507 of an Hogshead.

Example (5.)

Reduce $17^{\circ} : 44' : 19$ to the Decimal of a Sign of the Zodiac of 30°.

60 | 19 Seconds

60 | 44,316 Primes

30 | 17,73861 Degrees

Answer ,59128703 of a Sign of the Zodiac.

Example

Example (6.)

Reduce $7^{\prime} : 0^{\prime\prime} : 11^{\prime\prime\prime}$ a Duodecimal Fraction to the Decimal of a Foot.

$12 \mid 11$ Thirds

$12 \mid ,91\dot{6}$ Seconds

$12 \mid 7,0763\dot{8}$ Primes

Answ. $,589699\dot{0}7\dot{4}$ of a Foot.

CASE II.

Of Reduction.

How to reduce any Circulating Expression to its lowest possible Equivalent Vulgar Fraction.

RULE.

Find its Equivalent Single Fraction, as taught in *Art.* 12, and with this new Expression proceed, as in the Method of Reduction of Vulgar Fractions: So shall you obtain its lowest Equivalent Vulgar Fraction.

Example (1.)

Reduce $,\dot{5}7142\dot{8}$ to its lowest Equivalent Vulgar Fraction.

1st, $,\dot{5}7142\dot{8} = \frac{571428}{999999}$ its E. V. F. Which Expression being reduced by the Method of Vulgar Fractions, will produce $\frac{4}{7}$ its lowest Equivalent Vulgar Fraction.

Example

Example (2.)

Reduce $,38\dot{6}\dot{3}$ to its loweſt Equivalent V. F.

1ſt, $,38\dot{6}\dot{3} = \frac{3825}{9900}$ its Equivalent V. F.

And $\frac{3825}{9900} = \frac{17}{44}$ its loweſt E. V. F.

Example (3.)

Reduce $3,6\dot{4}2857\dot{1}$ to its loweſt Equivalent V. F.

1ſt, $3,6\dot{4}2857\dot{1} = \frac{36428535}{9999990}$ its Equivalent V. F.

And $\frac{36428535}{9999990} = \frac{51}{14}$ its loweſt E. V. F.

I have omitted their Operations at large, becauſe every Perſon ſkilled in Vulgar Fractions muſt know the Method of finding the greateſt common Meaſure to any two given Numbers.

CASE III.

Of Reduction.

How to find the Value of any Circulating Decimal, which expreſſes ſome known Part or Parts of that Integer, to which it refers, whether it be to Money, Weights, Time, or Meaſures, *&c.*

RULE.

Multiply the given Expreſſion, (according to the Laws of Circulating Numbers) by the Number of Units contained in the next lower Denomination of that Species, to which the given Expreſſion refers; and ſo proceed to multiply

tiply by its next lower Denominations, until you come to its lowest Parts: and the several Products shall be the several Parts required.

1st, Of Coin.

Example. (1.)

Reduce ,8739583 to the known Parts of a *L.* Sterling.

20

S. 17,479166

12

D. 5,75000

4

Qrs. 3,00

Answer 17 S. : 5 $\frac{3}{4}$ D.

Example (2.)

Reduce ,59920634 to the known Parts of a Guinea Sterling.

21

59920634

1198412698

S. 12,58333333

12

D. 7,000

Answer. 12 S. : 7 D.

Example (3.)

Reduce ,49074 to the known Parts of a Moidore of 27 *S*.

27

343518

981481

S. 13,24999

12

D. 3,000

Answer 13 S. : 3 D.

But to contract the Operations, observe, When any Multiplier greater than 12 is composed of two Digits, either with or without 0's; or of 12's, and some other Digits, either with or without 0's; then multiply by the Multipliers composed Numbers alternately, and the last Result shall be the Product sought.

Example (2.) *resumed.*

Reduce ,59920634 to the known Parts of a Guinea Sterling.

7

4,19444444

3

S. 12,583

12

D. 7,000

Answer 12 S. : 7 D.

Example

Example (3.) *resumed.*

Reduce $,49\dot{0}7\dot{4}$ to the known Parts of a Moidore of 27 *S.*

$$
\begin{array}{rl}
 & ,49\dot{0}7\dot{4} \\
 & \underline{\quad\quad 9} \\
 & 4,41\dot{6}66 \\
 & \underline{\quad\quad 3} \\
S. & 13,25\dot{0} \\
 & \underline{\quad 12} \\
D. & 3,\dot{0}0
\end{array}
$$

Answer S. 13 : D. 3

In all Results where the Repetend consists of some other Repetend of a fewer Number of Places of Figures, retain the latter only as above.

Example (4.)

Reduce $,\dot{5}$ to the known Parts of a *L.* Sterling.

$$
\begin{array}{rl}
 & \underline{\quad 4} \\
 & 2,\dot{2} \\
 & \underline{\quad 5} \\
S. & 11,\dot{1} = \dot{5}\times 20 \\
 & \underline{\quad 12} \\
D. & 1\ ,\dot{3} \\
 & \underline{\quad 4} \\
Q^{rs.} & 1,\dot{3}
\end{array}
$$

Answ. S. 11 : D. 1 : Qrs. $1\frac{1}{3}$ exact.

Example

Example (5.)

Reduce ,8984375 to the known Parts of a Mark of S. 13 : 4.

```
      13,3
9 | 2,6953125 = C. P. × 10.

     ,29947916
     26953125
     8984375
   ____________
S.  11,97916666
           12
   ____________
D.  11,75000
        4
   ____________
Qrs. 3,00
```

	S.	D.
Answer	11 :	$11\frac{3}{4}$

2dly, *Of* WEIGHTS.

Example (1.)

Reduce ,571428 to the known Parts of a Tun Averdupois.

```
        ,571428
              5
      _________
       2,857142
              4
      _________
Cwt.  11,428571
              4
      _________
Q      1,714285
              7
      _________
       4,999999
        4
      _________
lb.   20,0
```

	Cwt.	Qrs.	lb.
Answer	11 :	1 :	20

Value ,5953692053571428 of a Tun.

	Cwt.	Qrs.	lb.	Ozs.	
Answer	11 :	3 :	17 :	11	exact.

Example (2.)

Reduce $,45\dot{3}902\dot{4}$ to the known Parts of a Tun Averdupois.

$$
\begin{array}{lr}
 & ,45\dot{3}902\dot{4} \\
 & 5 \\
\hline
 & 2,26\dot{9}512\dot{1} \\
 & 4 \\
\hline
C^{wt.} & 9,0\dot{7}804\dot{8}7 \\
 & 4 \\
\hline
Q^{r.} & 0,3\dot{1}219\dot{5} \\
 & 7 \\
\hline
 & 2,1\dot{8}536\dot{5} \\
 & 4 \\
\hline
\text{lb.} & 8,7\dot{4}146\dot{3} \\
 & 4 \\
\hline
 & 2,9\dot{6}585\dot{3} \\
 & 4 \\
\hline
O^{zs.} & 11,8\dot{6}341\dot{4} \\
 & 4 \\
\hline
 & 3,4\dot{5}365\dot{8} \\
 & 4 \\
\hline
\textit{Drams} & 13,8\dot{1}463\dot{4} \\
\hline
\end{array}
$$

	Cwt.	Qrs.	lb	Ozs.	Drams.
Answ.	9 :	0 :	8 :	11 :	$13,8\dot{1}463\dot{4}$

Example

Example (3.)

Reduce ,8958$\dot{3}$ to the known Parts of ℔ Troy.

$$
\begin{array}{rl}
 & 12 \\
\hline
O^{zs.} & 10,\dot{7}5000 \\
 & 20 \\
\hline
D^{wt.} & 15,00 \\
\hline
\end{array}
$$

Answer Ozs. 10 : Dwt. 15

Example (4.)

Reduce ,97$\dot{7}\dot{2}$ to the known Parts of a ℔ Troy.

$$
\begin{array}{rl}
 & 12 \\
\hline
O^{zs.} & 11,72\dot{7}\dot{2} \\
 & 4 \\
\hline
 & 2,\dot{9}\dot{0} \\
 & 5 \\
\hline
D^{wt.} & 14,\dot{5}\dot{4} \\
 & 6 \\
\hline
 & 3,\dot{2}\dot{7} \\
 & 4 \\
\hline
G^{rs.} & 13,\dot{0}\dot{9} \\
\hline
\end{array}
$$

Answ. Ozs. 11 : Dwt. 14 : Grains 13,$\dot{0}\dot{9}$

Thus you may proceed to find the known Parts of any Decimal Expression given. I shall therefore propose but two or three Examples more, and with them conclude this Rule, leaving their Operations to the Practice of the Learner.

3dly,

3dly, *Of* TIME.

Reduce ,9285714 to the known Parts of a Year of 365,25 Days.

	Days.	Hrs.	Min.	Seconds.
Answer	339 :	3 :	51 :	25,71428.

4thly, *Of* MEASURES.

How many Feet and Inches is ,972 of a Yard.

	Feet.	Inches.
Answer	2 :	11.

How many Poles, Yards, Feet is ,34469 of a Furlong.

	Poles.	Yards.	Foot.
Answer	13 :	4 :	1.

5thly, *Of* MOTION.

How many Degrees, Minutes, Seconds, is ,59128703 of a Sign of the Zodiac.

	Degrees.	Minutes.	Seconds.
Answer	17 :	44 :	19.

Note, 30 Degrees make one Sign of the Zodiac.

CHAP. VII.

Of Involution and Evolution of Circulating Numbers.

1st, Of INVOLUTION.

Its Definition and Rule.

28. THE continued Multiplication of any Quantity into itself is called *Involution*, or the manner of raising the several Powers of that Quantity. For Example:

29. If a Quantity be multiplied by itself, the Product is called its Square, or 2d Power; its 1st Power, or Root, being the given Quantity itself. That 2d Power being multiplied by its first Power, the Product is called its Cube, or 3d Power. And the 3d Power being multiplied by its 1st Power, that Product is called its Biquadrate, or 4th Power. Thus you may proceed on to raise what Power you please of any given Quantity, whether Finite, or Circulate.

A TABLE of Infinite Squares, proceeding from the several Infinite Expressions from $,\dot{1}$ to $,\dot{9}$ Inclusive.

Roots or 1st Powers.		Their Squares or 2d Powers.		
$,\dot{1}$	Now	$,\dot{1} \times ,\dot{1} = ,\dot{0}1234567\dot{9}$		= 81 —1
$,\dot{2}$	And	$,\dot{2} \times ,\dot{2} = ,\dot{0}4938271\dot{6}$		= 2025 —1
$,\dot{3}$		$,\dot{3} \times ,\dot{3} = ,\dot{1}$		
$,\dot{4}$		$,\dot{4} \times ,\dot{4} = ,\dot{1}9753c86\dot{4}$		
$,\dot{5}$		$,\dot{5} \times ,\dot{5} = ,\dot{3}0864197\dot{5}$		
$,\dot{6}$		$,\dot{6} \times ,\dot{6} = ,\dot{4}$		
$,\dot{7}$		$,\dot{7} \times ,\dot{7} = ,\dot{6}0493827\dot{1}$		
$,\dot{8}$		$,\dot{8} \times ,\dot{8} = ,\dot{7}9012345\dot{6}$		
$,\dot{9}$		$,\dot{9} \times ,\dot{9} = 1,0$		

Note,

Note, As ,012345679, is the Square of ,1, and the Square Root of ,1 is ,3, therefore ,012345679, is the 4th Power of ,3. And as ,197530864 is the Square of ,4, and the Square Root of ,4 is ,6, therefore ,197530864 is the 4th Power of ,6.

And that the Square of ,9 is equal to 1 or Unity, is evident from hence, That ,9 infinitely continued is equal to 1, as is demonstrated (in *Art*. 13.) And the Square of 1, is 1, therefore the Square of ,9 is alſo equal to 1.

And for the ſame Reaſon the Square of 9, is equal to 100; the Square of 99, is equal to 10000: And the Square of 999, is equal to 1000000, and ſo on.

And ſo likewiſe the Square of ,09 is equal to ,01; the Square of ,009 is equal to ,0001; and the Square of ,0009 is equal to ,000001, and ſo on.

And as the 2d, 3d, 4th, 5th, or 6th Powers, *&c.* of 1 are ſeverally 1 or Unity; ſo likewiſe the 2d, 3d, 4th, 5th, or 6th Powers, *&c.* of ,9 are ſeverally 1 or Unity.

Here follows a TABLE of the Infinite Cubes, proceeding from the several Infinite Expressions, from ,1 to ,9 Inclusive.

$9^3 = 3^6 = 729$

$9^3 \cdot 5^3 = 45^3$

$15^6 = 11390625$

Roots or 1st Powers.		*Their Cubes or 3d Powers.*
,1	Now ,1×,1×,1 =	,001371742112482853223593964334705075445816186556927297668038408779149519890260631 the Cube or 3d Power of ,1.
,2	And ,2×,2×,2 =	,010973936899862825788751714677640603566529492455418381344307270233196159122085048 the Cube of ,2.
,3	,3×,3×,3 =	,037 the Cube of ,3.
,4	,4×,4×,4 =	,087791495198902606310013717421124828532235939643347050754458161865569272976680384 the Cube of ,4.
,5	,5×,5×,5 =	,171467764060356652949245541838134430727023319615912208504801097393689986282578875 the Cube of ,5.
,6	,6×,6×,6 =	,296 the Cube of ,6.
,7	,7×,7×,7 =	,470507544581618655692729766803840877914951989026063100137174211248285322359396433 the Cube of ,7.

Roots

Roots or 1st Powers.

,8 ,8×,8×,8 = ,7023319615912208504801097
3936899862825788751714677
64060356652949245554183813
443072 the Cube of ,8.

,9 ,9×,9×,9 = 1,0 the Cube of ,9.

For as much as ,00137 &c. as above, is the 3d Power of ,1, and the Square Root of ,1 is ,3, therefore ,00137 &c. is the 6th Power of ,3.

And as ,08779 &c. as above, is the 3d Power of ,4, and the Square Root of ,4 is ,6, therefore ,08779 &c. is the 6th Power of ,6.

Whoever ſhould be inclined to raiſe the 4th Powers of ,1 ,2 ,4, ,5 ,7 and ,8, will find that each Circulating Expreſſion will conſiſt of 729 Places of Figures deep. And in raiſing the 2d or 3d Powers of Compound Circulates, we muſt frequently be content to take an approximate Power, inſtead of the exact one; which will very often conſiſt of ſome Hundreds, or ſome Thouſands of Figures deep.

However, as the 2d, 3d, 4th, and 5th Powers of ,3, and of ,6, are to be found with little or no trouble, I chuſe in this place to exhibit their Operations at large; that he, whoſe Curioſity ſhould prompt him, might know the ſhorteſt Method how to raiſe the like or higher Powers of ,1 ,2 ,4 ,5 ,7 or ,8, or of any Compound Repetends.

1ſt,

1st, Let $,\dot{3}$ be given to be involved to its 5th Power.

Operation.

$3^{-1} =$ $\quad ,\dot{3}$

$\quad ,\dot{3}$

$9 \,|\, 1,\dot{0}$

$9^{-1} =$ $\quad ,\dot{1}$ its Square, or 2d Power.

$\quad ,\dot{3}$

$9 \,|\, ,\dot{3}$

$27^{-1} =$ $\quad ,\dot{0}3\dot{7}$ its Cube, or 3d Power.

$\quad ,\dot{3}$

$9 \,|\, ,\dot{1}11$

$81^{-1} =$ $\quad ,\dot{0}1234567\dot{9}$ its Biquadrate, or 4th Power.

$\quad ,\dot{3}$

$9 \,|\, ,\dot{0}3\dot{7}037037$

$243^{-1} =$ $\quad ,\dot{0}0411522633744855967078189\dot{3}$ its Surfolid, or 5th Power. $= 3^{-5}$

And if this laſt Reſult be multiplied by $,\dot{3}$, and its Product divided by 9, the Quotient will be its 6th Power, and ſo you may proceed on to raiſe what Power of $,\dot{3}$ you are inclined to.

Obſerve the 4th Power of $,\dot{3}$ is the 2d Power of $,\dot{1}$.

2dly,

2dly, Let ,$\dot{6}$ be involved to its 5th Power.

Operation.

15^{-1} ,$\dot{6}$

,$\dot{6}$

9 | 4,$\dot{0}$

225^{-1} ,$\dot{4}$ its Square, or 2d Power.

,$\dot{6}$

9 | 2,$\dot{6}$

3375^{-1} ,$\dot{2}9\dot{6}$ its Cube, or 3d Power.

,$\dot{6}$

9 | 1,$\dot{7}77$

50625^{-1} ,$\dot{1}9753086\dot{4}$ its Biquadrate, or 4th Power.

,$\dot{6}$

9 | 1,$\dot{1}8\dot{5}185185$

759375^{-1} = ,$\dot{1}3168724279835390946502057\dot{6}$ its Surfolid, or 5th Power.

Obferve that the 4th Power of ,$\dot{6}$ is the 2d Power of ,$\dot{4}$.

I fhall add more Examples, *viz.* of finding the Squares, or 2d Powers, of Compound Circulates, Pure and Mixt; but fhall leave their higher Powers for others to inveftigate, whofe Inclination fhall lead them thereto.

Example

Example (1.)

Find the Square of $,\dot{3}\dot{6}$.

Operation.

$,\dot{3}\dot{6}$
36

$\dot{2}1\dot{8}$
$\dot{1}0\dot{9}0$

Note $,\dot{3}\dot{6} = \frac{36}{99}$

99	13	$,\dot{0}\dot{9}$	09	09	09	09	09	09	09	09	09	09	09
		13	22	31	40	49	58	67	76	85	94	03	12
											1	1	1

$,\dot{1}3\ 22\ 31\ 40\ 49\ 58\ 67\ 76\ 85\ 95\ 0\dot{4}\ 13\ 22$

Its Square is the circulating Expreſſion, as mark'd above.

After the like Method the Square of $,\dot{1}\dot{8}$ will be found to be $,\dot{0}33057851239669421487\dot{6}$: And the Square of $,\dot{7}\dot{2}$ will be found to be $,\dot{5}28925619834710743801\dot{6}$.

Or otherwiſe thus:

Foraſmuch as we have already obtained the Square of $,\dot{3}\dot{6}$, and that the Expreſſion $,\dot{1}\dot{8}$ is the one half of $,\dot{3}\dot{6}$, therefore the $\frac{1}{4}$ (that is $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$) of the Square of $,\dot{3}\dot{6}$ will give the Square of $,\dot{1}\dot{8}$.

And

And as the Expreſſion $,\dot{7}\dot{2}$ is twice or 2 Times that of $,\dot{3}\dot{6}$, therefore 4 times (that is $2 \times 2 = 4$) the Square of $,\dot{3}\dot{6}$ will give the Square of $,\dot{7}\dot{2}$.

For obſerve, There is the ſame Harmony ſubſiſting between Similar Infinite Powers and their Roots, (the Difference of the Number of Places of Figures ariſing in their ſeveral Powers excepted) as is found to be between Similar Integral, Mixt or Fractional, Finite Powers and their Roots. The former indeed are to be conſidered as Infinite Expreſſions, and the latter as Finite ones.

Hence then, if I want the Square of any Multiple of the Expreſſion $,\dot{3}\dot{6}$ as 3 . 4 . 5 . 6 . 7 times *&c.* I firſt ſquare the 3 or 4 or 5 or 6 or 7 *&c.* and with it multiply the Square of $,\dot{3}\dot{6}$, according to the Laws of Multiplication: The Product ariſing ſhall be the Square required.

But if I wanted the Square of ſome aliquot Part of $,\dot{3}\dot{6}$, as its $\frac{1}{2} . \frac{1}{3} . \frac{1}{4} . \frac{1}{6} . \frac{1}{9}$ *&c.* I firſt ſquare the $\frac{1}{2}$ or $\frac{1}{3}$ or $\frac{1}{4}$ or $\frac{1}{6}$ or $\frac{1}{9}$ *&c.* and with it divide the Square of $,\dot{3}\dot{6}$, according to the Laws of Diviſion: the Quotient ariſing ſhall be the Square required. And not only the aliquot Parts, but the aliquant Parts thereof alſo might be taken too; but frequently it would prove a tedious Operation; ſuch as $\frac{1}{13}\ \frac{5}{17}\ \frac{11}{19}$ *&c.* Or ſuch Expreſſions as thoſe $1\frac{3}{29} \cdot 8\frac{5}{13} \cdot 7\frac{3}{17}$ *&c.*

Note, The ſame Harmony ſubſiſts in Similar Cubes, and in all higher Similar Powers; only there the Multiples, or

Aliquot Parts, muſt be cubed, &c. to obtain the Expreſſions required.

More Examples.

The Square of ,142857 is the circulating Expreſſion under $\frac{1}{49}$ (*viz.*) ,02040816 &c. *See* the Expreſſion at large in the Table.

$$\text{And} \left.\begin{matrix} 4 \\ 9 \\ 16 \\ 25 \\ 36 \end{matrix}\right\} \text{times the ſame Expreſſion is the Sq. of} \left\{\begin{matrix} ,285714 \\ ,428571 \\ ,571428 \\ ,714285 \\ ,857142 \end{matrix}\right.$$

The Square of ,54 is ,2975206611570247933884.

The Square of ,037, is the Cube of ,1. *Vide* the Table.

The Square of ,360 is ,1298595893190487785082379676974271568866163460758055352649947244541839136433731028325622992021751481210940670400.

The Square of ,63 is ,4049586776859504132231.

Find

Find the Square of $,1\dot{6}$.

Multiply $,1\dot{6}$ Note $,1\dot{6} = \frac{15}{90}$

by $,1\dot{6}$

Subst. 1

15 New Multiplier.

$8\dot{3}$

$1\dot{6}6$

$90 \mid 2,5\dot{0}$

$,02\dot{7}$ *Answer* $,02\dot{7}$.

Find the Square of $8,\dot{3}$.

Multiply $8,\dot{3}$

by $8,\dot{3}$

Subst. 8

75 New Multiplier.

$41\dot{6}$

$58\dot{3}3$

$9 \mid 625,\dot{0}$

$69,\dot{4}$ *Answer* $69,\dot{4}$.

Find the Square of $54,\dot{6}\dot{3}$.

Multiply	$54,\dot{6}\dot{3}$
by	$54,\dot{6}\dot{3}$
Subst.	54
	5409 New Multiplier.
	$491\dot{7}\dot{2}$
	000
	$2\dot{1}\dot{8}5454$
	$273\dot{1}\dot{8}181$

99	29	55	28,	$\dot{0}\dot{9}$	09	09	09	09	09	09	09	09	09	09	09	09	09
		29	84	12	21	30	39	48	57	66	75	84	93	02	11	20	29
			1	1	1	1	1	1	1	1	1	1	2	2	2	2	2

$29\dot{8}5,1322\,31\,40\,49\,58\,67\,76\,85\,95\,\dot{0}4\,13\,22\,31\,40$

Answer, Its Square is the Compound Mixt Circulate, as mark'd above.

Find

Find the Square of $,\dot{0}2\dot{7}$.

Multiply $,\dot{0}2\dot{7}$ Note the Expreſſion $,\dot{0}2\dot{7} = \frac{27}{999}$

by $,\dot{0}2\dot{7}$

Therefore 27 is the New Multiplier.

$\dot{1}8\dot{9}$

$0\dot{5}4\dot{0}$

$,\dot{7}2\dot{9}$

$\dot{9}9\dot{9}$	,729	729	729	729	729	729	729 &c.
		729	458	187	916	645	374 &c.
		1	2	2	3	4	5

$,\dot{0}00$ 730 460 189 919 649 379 108 8385682980277
5748721694667640613586559532505478451424397370
3433162892622352081811154127$\dot{1}$ the Square of $,\dot{0}2\dot{7}$.

Obſerve, That if you have the 2d, 3d, 4th, 5th, or 6th Power, &c. of any Root, or 1ſt Power, it is very eaſy to let the ſame Expreſſion, with a little Alteration, repreſent the Powers of the like Root, or 1ſt Power, by ſuppoſing its 1ſt Power to begin in the next Place, either higher or lower, in the Integral or Decimal Places; and that too by only removing the Decimal Diſtinction either two, three, or four Places of Figures, towards the right Hand:

Or elſe by prefixing 00, or 000, or 0000's, and placing the Decimal Diſtinction before the Whole, according as the Expreſſion is either a 2d, or 3d, or 4th Power, &c. And after that manner you may remove its Root, or 1ſt Power, to what Degree higher or lower you pleaſe.

For

For Instance; $,\dot{4}$ is the Square, or 2d Power of $,\dot{6}$; which Root, or 1st Power, begins at the Place of Tenths of an Unit. Now to have the Root $\dot{6}$ to begin in the Place of Units, or Tens, or Hundreds, &c. their several Squares will be thus; $\dot{4}4$, for the Root to begin to repeat in the Place of Units; and $\dot{4}444$, in the Place of Tens; and $\dot{4}44444$, in the Place of Hundreds, &c.

And on the contrary, to have the Root $\dot{6}$ to begin in the Place of Hundredths, or Thousandths, or Tens of Thousandths of an Unit, &c. their several Squares must be $,00\dot{4}$ and $,0000\dot{4}$ and $,000000\dot{4}$ &c.

So likewise $,\dot{0}3\dot{7}$ is the Cube, or 3d Power of $,\dot{3}$; which Root, or 1st Power, begins at the Place of Tenths of an Unit. Now to have the Root $\dot{3}$ to begin in the Place of Units, or Tens, or Hundreds, &c. their several Cubes will be $\dot{0}3\dot{7}$, and $\dot{0}3703\dot{7}$, and $\dot{0}3703703\dot{7}$, &c.

And on the contrary, to have the Root $\dot{3}$ to begin in the Place of Hundreths, or Thousandths, or Tens of Thousandths of an Unit, &c. their several Cubes must be $,000\dot{0}3\dot{7}$ and $,000000\dot{0}3\dot{7}$ and $,000000000\dot{0}3\dot{7}$.

And after this manner we may proceed with any Powers, so as to make their Roots to begin any where, higher or lower at Pleasure, regard being had to the different Alterations of their different Periods.

Many more Observations might be made concerning the Powers I have here exhibited, and their Roots, &c. but I am persuaded

persuaded that I have said enough; so that no Person can possibly be at a Loss, how to raise any Power his Patience will give him leave.

And by what hath been already said in this *Chapter* it is very evident, that an Infinite Number of Powers might be raised, which at first view one might take for Irrational or Surd Quantities, but which will have in their Roots the same Numbers again returning in a continual Circulation; as appears in the Interminate Quotient of a Division: and consequently that then their Roots will consist of a Mathematical exact Answer, and be as correct an Expression, as is the Root of any Rational Finite Number whatever. Wherefore I shall go on to

2*dly*, EVOLUTION, *or the Extraction of Roots.*

30. *Definitions.* Evolution is the Converse of Involution, and is the Art of finding from a given 2d, 3d, 4th, or 5th Power, *&c.* its Root, or 1st Power; which being involved, will produce its given Power, or be infinitely near it.

31. All Powers above the 1st are either Rational or Irrational.

32. Rational Powers are such Expressions as have their Roots capable of being expressed either by some Finite, or Circulating Expression.

33. Irrational Powers are such Expressions as have no such real Roots, that can be expressed either by any Finite, or Circulating Expression: Or at least have no such Roots, which are required to be extracted from them.

The Former chiefly will be the Subject of the ensuing Discourse.

The

The Method of extracting the Roots of Circulating Powers is the same with that of other Numerical Powers; care being taken in the Disposition of the several Periods by applying them alternately (like as in Division) to each new Resolvend as long as the Process is continued.

It is beside my intended Brevity to lay down Rules, or Canons, in this Place; for the Resolution of Powers: I therefore must refer the Reader to consult other Books on that Subject. And I suppose I cannot send him to a better, than to the Ingenious *Mr.* Ward's *Young Mathematician's Guide.*

EXAMPLES in the Square Root.

(1.) What's the Square Root of ,012345679?

It would be needless to exhibit the several Operations at large, my Reader being supposed to be throughly acquainted with the Method of Extractions; therefore I shall only express their Preparations with their Roots as follows.

The given Resolvend Prepared.

,0123456790 *&c.* (,11111 *&c.* its Root.

(2.) What's the Square Root of ,4?

Preparation.

,444444 *&c.* (,666 *&c.* its Root.

(3.) What's the Square Root of ,132231 *&c?* Vide *page* (138.)

Preparation.

,132231404958 *&c.* (,363636 *&c.* its Root.

(4.)

(4.) What's the Square Root of ,0013717 *&c.* Vide *the Cube of* 1.

Preparation.

,001371742112 *&c.* (,037037 *&c.* its Root.

(5.) What's the Square Root of ,1298595 *&c?* Vide *page* 140.

Preparation.

,129859589319 *&c.* (,360360 *&c.* its Root.

(6.) What's the Square Root of ,027 ?

Preparation.

,02702702 *&c.* (,1666 *&c.* its Root.

(7.) What's the Square Root of ,0204081 *&c?* Vide $\frac{1}{49}$ *in the Table.*

Preparation.

,020408163265 *&c.* (,142857 *&c.* its Root.

(8) What's the Square Root of 69,4 ?

Preparation.

,69,444444 *&c.* (8,333 *&c.* its Root.

(9.) What's the Square Root of 2985,132 &c? Vide *page* 142.

Preparation.

,2985,13223140 &c. (54,6363 &c. its Root.

EXAMPLES *in the Cube Root.*

(1.) What's the Cube Root of ,00137171 &c? Vide *Table of Cubes.*

Preparation.

,001371742112 &c. (,1111 &c. its Root.

(2.) What's the Cube Root of ,037?

Preparation.

,037037037037 &c. (,3333 &c. its Root.

(3.) What's the Cube Root of ,70233196 &c? Vide *Table of Cubes.*

Preparation.

,702331961591 &c. (,8888 &c. its Root.

EXAMPLES in the Biquadrate Root, or Square squared Root.

(1.) What's the Biquadrate Root of ,012345679? Vide *Table of Squares.*

RULE.

First extract the Square Root of the given Resolvend; and then the Square Root of its Root will be its Biquadrate Root required.

Preparation 1.

,0123456790 *&c.* (,11111 *&c.* its first Root.

Preparation 2.

,1111111111 *&c.* (,33333 *&c.* its Biquadrate Root.

(2.) What's the Biquadrate Root of ,197530864 *&c*?

Preparation 1.

,1975308641 *&c.* (,44444 *&c.* its first Root.

Preparation 2.

,44444444 *&c.* (,6666 *&c.* its Biquadrate Root.

EXAMPLES in the Surſolid Root, or Examples having the 5th Power given to find its Root.

(1.) What's the Surſolid Root of ,00411 *&c?* Vide *page* 136.

Preparation.

,004115226337448 *&c.* (,333 *&c.* its Root.

(2.) What's the Surſolid Root of ,13168724 *&c?* Vide *page* 137.

Preparation.

,131687242798353 *&c.* (,666 *&c.* its Surſolid Root.

EXAMPLES in the Square Cubed, or Cube Squared Root; or Examples having the 6th Power given to find its Root.

RULE.

Firſt Extract the Square Root of the given Reſolvend; and then the Cube Root of its Root will be the Cube Squared Root required.

(1.) What's the Cube Squared Root of ,00137 *&c?* Vide *Table of Cubes.*

Preparation 1.

,00137174211248285 3 *&c.* (,037037037 *&c.* its Square Root.

Preparation.

Preparation 2.

,037037037 *&c.* (,333 *&c.* its Cube Squared Root.

(2.) What's the Cube Squared Root of ,08779149 *&c?* Vide *Table of Cubes.*

Preparation 1.

,08779149519890260б *&c.* (,296296296 *&c.* its Squared Root.

Preparation 2.

,296296296 *&c.* (,666 *&c.* its Cube Squared Root.

Hitherto I have treated of Rational Powers only; and what Irrational Powers are, hath been already defined. I ſhall only add this, That for their Roots we muſt be content to give an Approximate Anſwer, inſtead of a Mathematical exact one. For inſtance; if it were required to extract the Square, or Cube, or Biquadrate, or Surſolid Root, *&c.* from ,142857 an Irrational Power; I ſay, we muſt be content to give an Approximate Anſwer for each of its Roots; but which will approach nearer and nearer the Truth, according as each Proceſs, by continually applying the given Circulate, is carried down lower and lower.

And though we cannot poſſibly come at its juſt Root, yet we may, by carrying on the Work, attain the Root ſo near the Truth, that its Defect ſhall be as little, or indeed leſs than any aſſignable Difference.

CONCLUSION.

I AM thoroughly perſuaded I need make no Apology to Men of my own Profeſſion for the Multitude of Examples exhibited in each Chapter; becauſe they muſt with me be fully convinced, how much more prevalent Examples are with their Pupils, than Precepts. For Youth indeed very ſeldom give a proper and careful Attention to the latter, whilſt by a Multitude of the former they will generally turn out ready practical Arithmeticians.

Wherefore I flatter myſelf, that even every conſiderate and ingenuous Reader will readily excuſe it; more eſpecially when he ſhall reflect that my Writing was chiefly deſigned to inform the weakeſt Capacities.

As to Perſons of a clear Head in Numerical Calculations, though ignorant of this Science, and yet deſirous to learn it, I am inclined to believe that leſs than the one fourth Part, of what I have here exhibited, would have been ſufficient for their perfect Information: To ſuch therefore I leave it to chuſe and reject at their own Diſcretion.

TABLES
OF
Equivalent Decimal Expressions
FOR ALL
FRACTIONS,
From the $\frac{1}{2}$ to the $\frac{98}{99}$ of an UNIT, &c.
PART. I.

DENOMINATORS.

NUMERATORS.	2	3	4	5
1	,5	,3	,25	,2
2		,6	,5	,4
3			,75	,6
4				,8
5				
6				
7				
8				

NUMERATORS.	10	11	12	13
1	,1	,09	,083	,076923
2	,2	,18	,16	,153846
3	,3	,27	,25	,230769
4	,4	,36	,3	,307692
5	,5	,45	,416	,384615
6	,6	,54	,5	,461538
7	,7	,63	,583	,538461
8	,8	,72	,6	,615384
9	,9	,81	,75	,692307
10		,90	,83	,769230
11			,916	,846153
12				,923076

DENOMINATORS.

NUMERATORS.	6	7	8	9
1	,16	,142857	,125	,1
2	,3	,285714	,25	,2
3	,5	,428571	,375	,3
4	,6	,571428	,5	,4
5	,83	,714285	,625	,5
6		,857142	,75	,6
7			,875	,7
8				,8

NUMERATORS.	14	15	16	17
1	,0714285	,06	,0625	,0588235294117647
2	,142857	,13	,125	
3	,2142857	,2	,1875	
4	,285714	,26	,25	
5	,3571428	,3	,3125	
6	,428571	,4	,375	
7	,5	,46	,4375	
8	,571428	,53	,5	
9	,6428571	,6	,5625	
10	,714285	,6	,625	
11	,7857142	,73	,6875	
12	,857142	,8	,75	

DENOMINATORS.

NUMERATORS.

	10	11	12	13
13				
14				
15				
	18	19	20	21
1	,05	,052631578947368421	,05	,047619
2	,1		,1	,095238
3	,16		,15	,142857
4	,2		,2	,190476
5	,27		,25	,238095
6	,3		,3	,285714
7	,38		,35	,3
8	,4		,4	,380952
9	,5		,45	,428571
10	,5		,5	,476190
11	,61		,55	,523809
12	,6		,6	,571428
13	,72		,65	,619047
14	,7		,7	,6
15	,83		,75	,714285
16	,8		,8	,761904
17	,94		,85	,809523

NUME-

DENOMINATORS.

NUMERATORS.	14	15	16	17
13	,9285714	,86	,8125	
14		,93	,875	
15			,9375	

NUMERATORS.	22	23	24	25
1	,045	,0434782608695652173913	,0416	,04
2	,09		,083	,08
3	,136		,125	,12
4	,18		,16	,16
5	,227		,2083	,2
6	,27		,25	,24
7	,318		,2916	,27
8	,36		,3	,32
9	,409		,375	,36
10	,45		,416	,4
11	,5		,4583	,44
12	,54		,5	,48
13	,590		,5416	,52
14	,63		,583	,56
15	,681		,625	,6
16	,72		,6	,64
17	,772		,7083	,68

DENOMINATORS.

NUMERATORS.	18	19	20	21
18			,9	,857142
19			,95	,904761
20				,952380
21				
22				
23				
24				

NUMERATORS.	26	27	28	29
1	,0384615	,037	,03571428	,0344827586206896551724137931
2	,076923	,074	,0714285	
3	,1153846	,1	,10714285	
4	,153846	,148	,142857	
5	,1923076	,185	,17857142	
6	,230769	,2	,2142857	
7	,2692307	,259	,25	
8	,307692	,296	,285714	
9	,3461538	,3	,32142857	
10	,384615	,370	,3571428	
11	,4230769	,407	,39285714	
12	,461538	,4	,428571	
13	,5	,481	,46428571	

NUME-

DENOMINATORS.

NUMERATORS.	22	23	24	25
18	,81		,75	,72
19	,863		,7916	,76
20	,90		,83	,8
21	,954		,875	,84
22			,916	,88
23			,9583	,92
24				,96

NUMERATORS.	30	31	32	33
1	,03	,0322580645161290	,03125	,03
2	,06		,0625	,06
3	,1		,09375	,09
4	,13		,125	,12
5	,16		,15625	,15
6	,2		,1875	,18
7	,23		,21875	,21
8	,26		,25	,24
9	,3		,28125	,27
10	,3		,3125	,30
11	,36		,34375	,33
12	,4		,375	,36
13	,43		,40625	,39

NUME-

DENOMINATORS.

NUMERATORS.	26	27	28	29
14	,538461	,518	,5	
15	,5769230	,5	,53571428	
16	,615384	,592	,571428	
17	,6538461	,629	,60714285	
18	,692307	,6	,6428571	
19	,7307692	,703	,67857142	
20	,769230	,740	,714285	
21	,8076923	,7	,75	
22	,846153	,814	,7857142	
23	,8846153	,851	,82142857	
24	,923076	,8	,857142	
25	,9615384	,925	,89285714	
26		,962	,9285714	
27			,96428571	
28				
29				
30				
31				
32				

NUME-

DENOMINATORS.

NUMERATORS.	30	31	32	33
14	,4$\dot{6}$		,4375	,$\dot{4}\dot{2}$
15	,5		,46875	,$\dot{4}\dot{5}$
16	,5$\dot{2}$		,5	,$\dot{4}\dot{8}$
17	,5$\dot{6}$		,53125	,$\dot{5}\dot{1}$
18	,6		,5625	,$\dot{5}\dot{4}$
19	,6$\dot{3}$		,59375	,$\dot{5}\dot{7}$
20	,$\dot{6}$		,625	,$\dot{6}\dot{0}$
21	,7		,65625	,$\dot{6}\dot{3}$
22	,7$\dot{3}$		,6875	,$\dot{6}\dot{6}$
23	,7$\dot{6}$		,71875	,$\dot{6}\dot{9}$
24	,8		,75	,$\dot{7}\dot{2}$
25	,8$\dot{3}$		,78125	,$\dot{7}\dot{5}$
26	,8$\dot{6}$		,8125	,$\dot{7}\dot{8}$
27	,9		,84375	,$\dot{8}\dot{1}$
28	,9$\dot{3}$		,875	,$\dot{8}\dot{4}$
29	,9$\dot{6}$		,90625	,$\dot{8}\dot{7}$
30			,9375	,$\dot{9}\dot{0}$
31			,96875	,$\dot{9}\dot{3}$
32				,$\dot{9}\dot{6}$

DENOMINATORS.

NUMERATORS.	34	35	36	37	38
1	,02941176470588235	,0285714	,027	,027	,0263157894736842105
2		,0571428	,05	,054	
3		,0857142	,083	,081	
4		,1142857	,1	,108	
5		,142857	,138	,135	
6		,1714285	,16	,162	
7		,2	,194	,189	
8		,2285714	,2	,216	
9		,2571428	,25	,243	
10		,285714	,27	,270	
11		,3142857	,305	,297	
12		,3428571	,3	,324	
13		,3714285	,361	,351	
14		,4	,38	,378	
15		,428571	,416	,405	
16		,4571428	,4	,432	
17		,4857142	,472	,459	
18		,5142857	,5	,486	
19		,5428571	,527	,513	
20		,571428	,5	,540	
21		,6	,583	,567	

NUME-

DENOMINATORS.

NUMERATORS.	39	40	41	42
1	,025641	,025	,02439	,0238095
2	,051282	,05	,04878	,047619
3	,076923	,075	,07317	,0714285
4	,102564	,1	,09756	,095238
5	,128205	,125	,12195	,1190476
6	,153846	,15	,14634	,142857
7	,179487	,175	,17073	,16
8	,205128	,2	,19512	,190476
9	,230769	,225	,21951	,2142857
10	,256410	,25	,24390	,238095
11	,282051	,275	,26829	,2619047
12	,307692	,3	,29268	,285714
13	,3	,325	,31707	,3095238
14	,358974	,35	,34146	,3
15	,384615	,375	,36585	,3571428
16	,410256	,4	,39024	,380952
17	,435897	,425	,41463	,4047619
18	,461538	,45	,43902	,428571
19	,487179	,475	,46341	,4523809
20	,512820	,5	,48780	,476190
21	,538461	,525	,51219	,5

DENOMINATORS.

NUMERATORS.	34	35	36	37	38
22		,6285714	,61	,594	
23		,6571428	,638	,621	
24		,6857142	,6	,648	
25		,714285	,694	,675	
26		,7428571	,72	,702	
27		,7714285	,75	,721	
28		,8	,7	,756	
29		,8285714	,805	,783	
30		,857142	,83	,810	
31		,8857142	,861	,837	
32		,9142857	,8	,864	
33		,9428571	,116	,891	
34		,9714285	,94	,918	
35			,972	,945	
36				,972	
37					
38					
39					
40					
41					

DENOMINATORS.

NUMERATORS.	39	40	41	42
22	,564102	,55	,53658	,523809
23	,589743	,575	,56097	,5476190
24	,615384	,6	,58536	,571428
25	,641025	,625	,60975	,5952380
26	,6	,65	,63414	,619047
27	,692307	,675	,65853	,6428571
28	,717948	,7	,68292	,6
29	,743589	,725	,70731	,6904761
30	,769230	,75	,73170	,714285
31	,794871	,775	,75609	,7380952
32	,820512	,8	,78048	,761904
33	,846153	,825	,80487	,7857142
34	,871794	,85	,82926	,809523
35	,897435	,875	,85365	,83
36	,923076	,9	,87804	,857142
37	,948717	,925	,90243	,8809523
38	,974358	,95	,92682	,904761
39		,975	,95121	,9285714
40			,97560	,952380
41				,9761904

DENOMINATORS.

NUMERATORS.	43	44	45	46
1	,023255813953488372093	,0227	,02	,02173913043478260869565
2		,045	,04	
3		,0681	,06	
4		,09	,08	
5		,1136	,1	
6		,136	,13	
7		,1590	,15	
8		,18	,17	
9		,2045	,19	
10		,227	,2	
11		,25	,24	
12		,27	,26	
13		,2954	,28	
14		,318	,31	
15		,3409	,3	
16		,36	,35	
17		,3863	,37	
18		,409	,39	
19		,4318	,42	
20		,45	,4	

NUME-

DENOMINATORS.

NUMERATORS.	47	48	49	50
1	,0212765957446808510638297872340425531914893617	,02083	,020408163265306122448979591836734693877551	,02
2		,0416		,04
3		,0625		,06
4		,083		,08
5		,10416		,1
6		,125		,12
7		,14583		,14
8		,16		,16
9		,1875		,18
10		,2083		,2
11		,22916		,22
12		,25		,24
13		,27083		,26
14		,2916		,28
15		,3125		,3
16		,3		,32
17		,35416		,34
18		,375		,36
19		,39583		,38
20		,416		,4

DENOMINATORS.

NUMERATORS.	43	44	45	46
21		,4772	,46	
22		,5	,48	
23		,5227	,51	
24		,54	,53	
25		,5681	,5	
26		,590	,57	
27		,6136	,6	
28		,63	,62	
29		,6590	,64	
30		,681	,6	
31		,7045	,68	
32		,72	,71	
33		,75	,73	
34		,772	,75	
35		,7954	,7	
36		,81	,8	
37		,8409	,82	
38		,860	,84	
39		,8863	,86	
40		,90	,8	

DENOMINATORS.

NUMERATORS.	47	48	49	50
21		,4375		,42
22		,4583		,44
23		,47916		,46
24		,5		,48
25		,52083		,5
26		,5416		,52
27		,5625		,54
28		,583		,56
29		,60416		,58
30		,625		,6
31		,64583		,62
32		,6		,64
33		,6875		,66
34		,7083		,68
35		,72916		,7
36		,75		,72
37		,77083		,74
38		,7916		,76
39		,8125		,78
40		,83		,8

DENOMINATORS.

NUMERATORS.	43	44	45	46
41		,93$\dot{1}\dot{8}$	,9$\dot{1}$	
42		,9$\dot{5}\dot{4}$	,9$\dot{3}$	
43		,97$\dot{7}\dot{2}$	,9$\dot{5}$	
44			,9$\dot{7}$	
45				
46				
47				
48				
49				

DENOMINATORS.

NUMERATORS.	47	48	49	50
41		,8541$\dot{6}$		,82
42		,875		,84
43		,8958$\dot{3}$		,86
44		,91$\dot{6}$		,88
45		,9375		,9
46		,958$\dot{3}$		,92
47		,9791$\dot{6}$		,94
48				,96
49				,98

PART II.

	51	52	53	54
1	,0196078431372549	,01923076	,0188679245283	,185

	59	60	61	62
1	,0169491525423728813559322033898305084745762711864406779661	,016	,01639344262295081967213114754098360655737049180327868852459	,0161290322580645

	55	56	57	58
1	,018	,017857142	,017543859649122807	,0172413793103448 2758620689655

	63	64	65	66
1	,015873	,015625	,0153846	,015

	67	68	69	70
1	,01492537313432 8358208955223880597	,01470588235294 1176	,01449275362318 84057971	,0142857

	75	76	77	78
1	,013	,01315789473684 210526	,012987	,0128205

	83	84	85	86
1	,01204819277108433734939759036144578313253	,01190476	,01176470588235294	,0116279069767441860465

		71	72	73	74
	1	,01408450704225 352112676056338 028169	,0138	,01369863	,0135

		79	80	81	82
	1	,0126582278481	,0125	,012345679	,012195

		87	88	89	90
	1	,011494252873563218390804 5977	,01136	,0112359550561797752808 98 876404494382022471 91	,01

	91	92	93		94
1	,010989	,0108695652173913043478 26	,010752688172043	1	,0106382978723404255319148936170212 7659574468085

	99
1	,01

	95	96	97	98
1	,0105263157894736842	,010416	,0103092783505154639175257731958762 88659793814432989690721649484536082 47422680412371134020618 5567	,0102040816326530612244897959183673 469387755

The END *of the* TABLES.

THE

THE EXPLANATION and USE OF THE Foregoing TABLES.

Their EXPLANATION.

THE Tables are divided into two Parts. The firſt Part exhibits the Equivalent Decimal Expreſſions for all Fractions, (except ſuch whoſe Circulates run deep before they end) from the $\frac{1}{2}$ to the $\frac{49}{50}$ of an Unit incluſive; which are found by Inſpection only.

The ſecond Part, for the ſake of Brevity, exhibits only Tabular Numbers from $\frac{1}{51}$ $\frac{1}{52}$ $\frac{1}{53}$ *&c.* to $\frac{1}{99}$, with their ſeveral Equivalent Decimal Expreſſions.

Their USE.

Let it be required to find the Decimal Fraction equal to $\frac{7}{8}$.

Firſt find its Denominator 8 on the Top of the Tables; then in that Column right againſt 7, found among the Numerators in the ſide Column, you will find ,875; which is its Equivalent Decimal Fraction.

And

And after the like manner you will find that $\frac{5}{7} =$ $,\dot{7}1428\dot{5}$; And $\frac{1}{14} = ,0\dot{7}1428\dot{5}$; And $\frac{11}{12} = ,91\dot{6}$; And $\frac{20}{21} = ,\dot{9}5238\dot{0}$; And $\frac{16}{33} = ,\dot{4}\dot{8}$; And $\frac{15}{28} =$ $,53\dot{5}7142\ddot{8}$; And $\frac{1}{17} = ,\dot{0}58823$ &c. And $\frac{1}{49} =$ $,\dot{0}20408$ &c. And so on.

Observe, Where the Decimals run deep, as at $\frac{1}{17}$ $\frac{1}{19}$ $\frac{1}{23}$ $\frac{1}{29}$ $\frac{1}{31}$ $\frac{1}{34}$ &c. I there contented myself with placing in their several Columns the corresponding Decimals to each of them only; but any one, who is inclined, might easily find the Decimals answering to any of their Multiples, by multiplying either of them by the Numerators of their given Parts, according to the Laws of circulating Numbers. For instance; let the Decimal of $\frac{3}{17}$ be required. First find in the Tables the Decimal of $\frac{1}{17}$; then multiply that by 3; its Result shall be the Equivalent Decimal corresponding to $\frac{3}{17}$. And if that Number found in the Tables was multiplied by 4 . 5 . 6 . 7 . 8 . 9 or 10 &c. the several Results would be the Equivalent Decimals to $\frac{4}{17}$ $\frac{5}{17}$ $\frac{6}{17}$ $\frac{7}{17}$ $\frac{8}{17}$ $\frac{9}{17}$ or $\frac{10}{17}$ &c.

Again, Let the Decimal of $\frac{36}{49}$ be required. First find in the Tables the Decimal equal to $\frac{1}{49}$; then multiply that

 by

by 36; its Refult fhall be the Equivalent Decimal correfponding to $\frac{36}{49}$. Which Infinite Decimal will alfo be the Square, or 2d Power of $,\dot{8}5714\dot{2} = \frac{6}{7}$. For the Square Root of $\frac{36}{49} = \frac{6}{7}$; and $\frac{6}{7} = ,\dot{8}5714\dot{2}$.

It is moft commodious to let the Vulgar Fraction given, be expreffed in its leaft Terms, before you find its Equivalent Decimal one. And when it is fo reduced, if it appears that the given Fraction is any Aliquot Part of fome one Fraction in the foregoing Tables, we can from thence readily obtain its Equivalent Decimal Fraction. Thus;

Let the reduced Fraction be $\frac{7}{114}$. Here $\frac{7}{114}$ is the $\frac{1}{2}$ of $\frac{7}{57}$; wherefore firft find the Decimal Expreffion for $\frac{7}{57}$, then the $\frac{1}{2}$ of that fhall be the Decimal equal to $\frac{7}{114}$.

Again; Let the reduced Fraction be $\frac{113}{255}$. Here $\frac{1}{255}$ is the $\frac{1}{5}$ of $\frac{1}{51}$; wherefore firft find the Decimal Expreffion for $\frac{1}{51}$ by the Tables; then the 5th Part of that fhall be the Decimal equal to $\frac{1}{255}$; which being multiplied by 113, the given Numerator, this laft Refult fhall be the Decimal equal to $\frac{113}{255}$.

Hence

Hence then, by ſhewing the Uſe of the Tables, it is evident that we have obtained this farther Advantage, *viz.* a Method how to find the Equivalent Decimal Expreſſion to any Fraction, that is either a Mutiple, or an Aliquot Part or Parts of any one Fraction from the $\frac{1}{2}$ to the $\frac{98}{99}$ of an Unit incluſive, by the Aſſiſtance of the foregoing Tables.

It may perhaps be objected by ſome, That foraſmuch as large Circulates are not eaſily managed in Arithmetical Operations, therefore I might have ſaved myſelf the Trouble of particularly entering of them. To ſuch, I anſwer, That there being no univerſal Rule, that I know of, but by Way of Eſſay, to determine how many Places of Figures ſome Vulgar Fractions may require to compleat or form their Circulates, and having found thoſe which fall within the Compaſs of my Tables, I was willing to exhibit them there at large; that the Practitioner might the more readily perceive, in many Caſes, when it is moſt commodious to deal with Approximates.

I take the Liberty to add the following Table, becauſe I think it may be very acceptable to ſuch Perſons as would moſt correctly compute the Intereſt of any given Sum of Money, particularly large Sums, for any Number of Days. And for many other Reaſons, that might be aſſigned.

A TABLE for the ready finding the exact Decimal Parts of a Year equal to any Number of Days, &c.

Days.	Days.	Days.
1 =,002739726	10=,02739726	100= ,27397260
2 =,005479452	20=,05479452	200= ,54794520
3 =,008219178	30=,08219178	300= ,82191780
4 =,010958904	40=,10958904	365=1,0000000
5 =,01369863	50=,13698630	$\frac{1}{4}$ of a Year=,25
6 =,016438356	60=,16438356	
7 =,019178062	70=,19178062	$\frac{1}{2}$ of a Year=,5
8 =,021917808	80=,21917808	
9 =,024657534	90=,24657534	$\frac{3}{4}$ of a Year =,75

The USE of this TABLE *is thus:*

If the proposed Number of Days can be exactly found in the Table, (under Days) their exact Decimal Parts are also found against them by Inspection only.

But when the given Number of Days cannot be found there at one View, then both They, and their Decimal Parts must be collected out of the Table at twice, or thrice, according as the given Number requires.

As

As for *Example*: Suppose it were required to find the Decimal Parts of a Year equal to 299 Days.

$$\text{Then}\left\{\begin{array}{l} \overset{\text{Days}}{200} = ,\dot{5}479452\dot{0}5 \\ \;\;90 = ,\dot{2}465753\dot{4}2 \\ \;\;\;\;9 = ,0\dot{2}465753\dot{4} \end{array}\right\}$$

Add these Parts together according to the Laws of Circulating Decimals.

Hence $\underline{299 = ,8\dot{1}917808\dot{2}}$ the Decimal Parts required.

AN

AN

APPENDIX,

CONTAINING

The Arithmetic *of the Five primary* Rules *in* Decimal Fractions *as commonly Taught.*

CHAP. I.

A Fraction is a Part or Parts of an Unit.

A Decimal Fraction is when the Unit is supposed to be divided into Ten equal Parts, and each of those into Ten more equal Parts; and so descending in such Progression, that by a continual Decimal Subdivision, the Unit may be supposed to be divided into 10, or 100, or 1000, or 10000, or 100000, *&c.* Equal Parts, called 10ths, 100ths, 1000ths, 10000ths, 100000ths Parts of an Unit.

A Decimal Fraction is now frequently distinguished from whole Numbers, by prefixing a Comma before the Figure, or Figures, expressing the Decimal Fraction.

Thus ,5 called 5 Tenths of an Unit; and ,04 called 4 Hundredths of an Unit; and ,596 called 596 Thousandths of an Unit; and ,00703 called 703 One Hundred Thousandths of an Unit; and ,000055 called 55 Millionths of an Unit.

The

The Denominator of any Decimal Fraction is determined by Inspection only: for it must, agreeable to the Definition above, consist of an Unit, or 1, with as many o's annexed to it, as there are Places of Figures in the given Decimal Expression.

The following Table will best exhibit the Decimal Subdivisions of Unity, &c.

Integers.

7. X of Millions.
6. Millions.
5. C of Thousands.
4. X of Thousands.
3. Thousands.
2. C.
1. X.

UNITS.

Decimal Parts or Fractions.

1 Primes or Tenth Parts.
2 Seconds or C Parts.
3 Thirds or Thousandth Parts.
4 Fourths or X Thousandth Parts.
5 Fifths or C Thousandth Parts.
6 Sixths or Millionth Parts.
7 Sevenths or X Millionth Parts.

1*st*, The above Table shews, That the first Place, from the Place of Unity towards the Left-hand, is the Place of Tens; the second Place, the Place of Hundreds; and the third Place, the Place of Thousands, and so on, being Whole Numbers.

2*dly*, That

2*dly*, That the firſt Place, from the Place of Unity towards the Right-hand, is the Place of Tenths of an Unit; the ſecond Place, the Place of Hundreths of an Unit; and the third Place, the Place of Thouſandths of an Unit, and ſo on, being the ſeveral Decimal Subdiviſions of Unity.

I am perſuaded that a little Reflection will ſoon convince even every common Reader, that as the Places of Whole Numbers increaſe or decreaſe in a decuple, or tenfold Proportion, ſo do the Places of Decimal Expreſſions likewiſe increaſe or decreaſe, in a decuple, or tenfold Proportion.

Hence then it will naturally follow, that all the Operations in Addition, Subtraction, Multiplication, and Diviſion, of Finite Decimal Fractions, muſt in every Reſpect be the ſame with the Operations in Addition, Subtraction, *&c.* of whole Numbers.

I would have the Learner carefully obſerve, *viz.* That as the Expreſſion ,5 is equal to $\frac{5}{10} = \frac{50}{100} = \frac{500}{1000} = \frac{5000}{10000}$, and ſo on, however thus varied.

Or as the Expreſſion ,04 is equal to $\frac{4}{100} = \frac{40}{1000} = \frac{400}{10000} = \frac{4000}{100000}$, and ſo on, however thus varied.

So from hence it is manifeſt, that if to any given Decimal Expreſſion you annex any Number of o's at Pleaſure, it neither increaſes nor decreaſes the Value thereof.

Again, with Regard to Whole Numbers.

As $1 = \frac{10}{10} = \frac{100}{100} = \frac{1000}{1000} = \frac{10000}{10000}$ *&c.*

Or

$$\text{Or as } 2 = \frac{20}{10} = \frac{200}{100} = \frac{2000}{1000} = \frac{20000}{10000} \text{ \&c.}$$

$$\text{Or as } 50 = \frac{500}{10} = \frac{5000}{100} = \frac{50000}{1000} = \frac{500000}{10000} \text{ \&c.}$$

So likewise is it manifest, that if to any given Integral Number you annex any Number of o's, with their Decimal Distinction between it and them, the Integral Number will still continue of the same Value as before.

The Use and Advantage of annexing o's at pleasure, will appear in Subtraction and Division.

Dr. *Wallis* remarks, that the first Author who professedly treated of this Subject, was *Simon Stevinus*, in a Treatise (which he calls *Disme* or *Decimals*) subjoined to his Arithmetic, published in *French*, and printed at *Leyden*, (in *Christopher Plantin*'s Printing-House) in the Year 1585, which he had first written in *Dutch*, (and perhaps had published in that Language) and after translated into *French*, and so published it. Vide *History of Algebra*, Chap. 9.

This artificial Way of expressing any Part, or Parts of Unity, can never be too highly esteemed. For how wonderfully quick would all the Rules incident to Arithmetic be gone through, if universally all the various Weights, Coins, Measures, and Time, were thus Decimally subdivided. I am inclined to think, that three Months would be more than sufficient Time for One but of a tolerable Capacity, to turn out a compleat Arithmetician in. But how much soever one might heartily wish for, yet we cannot expect or hope to see so happy, and so uniform an Establishment. Therefore let us proceed to

CHAP. II.

ADDITION.

RULE.

BE careful to place Tenths under Tenths, Hundredths under Hundredths, and Thousandths under Thousandths, *&c.* as underneath: And then proceed to add up the several given Expressions, whether Simple, or Mixt, as in Addition of Whole Numbers; its Result, when marked off as below, will be the Total sought.

Examples.

1st, Simple.		2dly, Mixt.
(1.)	(2.)	(3.)
℔	*Yards.*	*Cwt.*
,5271	,159	174,5
,0714	,7	96,704
,1021	,1947	1,975
,0755	,98755	,4
,7761 Total	2,04125 Total	8,578635
		282,157635.

CHAP.

CHAP. III.

SUBTRACTION.

RULE.

PLACE Tenths under Tenths, Hundredths under Hundredths, &c. as before taught. Then proceed to ſubtract the given Expreſſions, whether Simple, or Mixt, as in Subtraction of Whole Numbers; the Reſult will be the Difference ſought.

Examples.

	Yards.		℔.		*Yards.*
From	,534	From	4,475965	From	39,4715
Take	,396	Take	,9975	Take	8,794765
Diff.	,138	Diff.	3,478465	Diff.	30,676735

CHAP. IV.

MULTIPLICATION.

RULE.

PROCEED with both Factors in every Reſpect as in Whole Numbers.

And to determine the Value of the Product, obſerve the following Directions.

1ſt, Mark off as many Places of Figures for the Fractional Part in the Product, as there are Decimal Places given in both Factors.

2*dly*, But when, as it may often happen, there are not so many Places of Figures in the Product, as there are Decimal Places given in both Factors, be careful to prefix as many 0's to the first imperfect Product, as is sufficient to supply the Defect. And then set the Decimal Distinction before the Whole Expression for the Product, as in the 3d, 4th, and 5th Examples following.

Examples.

(1.)

Multiply 47,583
by 5

Product 237,915

(2.)

Multiply 5,0125
by ,6

Product 3,00750

(3.)

Multiply ,175947
by ,0003

Product ,0000527841

(4.)

Multiply ,25
by ,25

125
50

Product ,0625

(5.)

Multiply ,57042
by ,079

513378
399294

Product ,04506318

(6.)

Multiply 57042
by ,079

513378
399294

Product 4506,318

CHAP.

CHAP. V.

DIVISION.

RULE.

PROCEED in the Operation, as in Whole Numbers. And to determine the Value of the Quotient, obſerve the following Directions.

1*ſt*, When the Diviſor, whether it be wholly Integral, or Mixt, or Simple, conſiſts of more Places of Figures than its given Dividend, be careful to annex o's at pleaſure to the Dividend, ſo as to continue the Operation until the Reſult in the Quotient may come as near the Truth as Neceſſity may require.

2*dly*, Mark off in the Quotient as many places of Figures for the Fractional Part, as is the Exceſs of the Decimal Places uſed in the Dividend, more than the Decimal Places in the Diviſor: That is, the Number of Decimal Places in the Quotient and Diviſor, muſt be equal to the Number of Decimal Places uſed in the Dividend.

3*dly*, When there are not Places of Figures enough in the Quotient to mark off for the Fractional Part, you muſt prefix a ſufficient Number of o's to the imperfect Quotient, to ſupply the Defect, and then ſet the Decimal Diſtinction before the whole Expreſſion, for the Quotient, as in the 3d, 4th, 8th, and 9th Examples following.

Examples.

(1.)	(2.)
7 | 4865,3471	,5 | 87,9471
Quote 695,0495 -|-	Quote 175,8942

(3.)

(3.)

8 | ,150472

Quote ,018809

(4.)

1,2 | ,0017544

Quote ,001462

(5.)

Divide ,475 by ,0012

,0012 | ,4750000

Quote 395,833 +

(6.)

Divide 1 by ,9

,9 | 1,000000

Quote 1,11111 +

(7.)

4,9 | 479,47585
441

Q. 97,8522 +
384 &c.

(8.)

D. 4 by 57,49

57,49 | 4,0000000
3 4494

Q. ,06957 +
55060 &c.

(9.)

Divide 1 by 48

48 | 1,000000
96

Quote ,020833 +
40 &c.

CHAP. VI.

REDUCTION.

TO reduce a Vulgar Fraction to its Equivalent Decimal Fraction, or near it.

RULE

RULE.

Divide the Numerator of the given Fraction, with a sufficient Number of o's annexed, by its Denominator; the Quotient will be the Decimal sought.

Demonstration.

The Reason is manifest. For the Proportion is,
As the Denominator of the given Fraction
is to its Numerator;

So is the proposed Denominator 10, or 100, or 1000, *&c.* to its Numerator sought.

Hence then it is evident, if the Numerator of the given Fraction be multiplied by 10, or 100, or 1000, *&c.* and that Product divided by the Denominator, the Quotient arising must be the New Numerator to its Denominator 10, or 100, or 1000, *&c.*

Examples.

(1.) Reduce $\frac{1}{2}$ to a Decimal Fraction.

$$\begin{array}{r|l} 2 & 1,0 \\ \hline & ,5 \end{array}$$

Answer ,5 Finite.

(2.) Reduce $\frac{3}{4}$ to a Decimal Fraction.

$$\begin{array}{r|l} 4 & 3,00 \\ \hline & ,75 \end{array}$$

Answer ,75 Finite.

(3.) Reduce $\frac{5}{8}$ to a Decimal Fraction.

$$\begin{array}{r|l} 8 & 5,000 \\ \hline & ,625 \end{array}$$

Answer ,625 Finite.

(4.) Re-

(4.) Reduce $\frac{3}{640}$ to a Decimal Fraction.

$$\begin{array}{r|l} 640 & 3,0000000 \\ ,0046875 & \underline{2\ 56} \\ & 44 \ \&c. \end{array}$$

Answ. ,0046875 Finite.

(5.) Reduce $\frac{7}{11}$ to a Decimal Fraction.

$$\begin{array}{r|l} 11 & 7,000000 \\ & ,636363 \ \&c. \end{array}$$

Answer ,636363 approximately. But where 63 would repeat infinitely in its Quotient.

(6.) Reduce $\frac{4}{21}$ to a Decimal Fraction.

$$\begin{array}{r|l} 21 & 4,000000 \\ ,190476 \ \&c. & \underline{21} \\ & 190 \ \&c. \end{array}$$

Answer ,190476 approximately. But where ,190476 would repeat infinitely in its Quotient.

(7.) Reduce $\frac{1}{960}$ to a Decimal Fraction: Or, in other words, find the Decimal Expression equal to One Farthing, a *L.* Sterling being the Integer.

$$\begin{array}{r|l} 960 & 1,0000000 \\ ,0010416 \ \&c. & \underline{96} \\ & 40 \ \&c. \end{array}$$

Answer ,0010416 approximately. But where 6 would repeat Infinitely from the Place of 10000000ths in its Quotient, if the Division was continued on *ad Infinitum.*

It

It may be an agreeable Amuſement to ſome of my Readers, when they are acquainted with the Management of Infinite Decimals, to find that the Decimal Expreſſion ,001041$\dot{6}$, being the Diviſor to any given Number of Pounds Sterling, will give in its Quotient their Equivalent Number of Farthings mathematically exact; and on the contrary, that the ſame Decimal Expreſſion, being multiplied by any given Number of Farthings, will give in its Product their equivalent Number of Pounds, *&c.*

I muſt farther obſerve, that from the above Expreſſion might be compoſed an accurate large Decimal Table of all the intermediate known Parts of a *L.* Sterling, by multiplying it according to the Laws of Circulates by 2, 3, 4, 5, *&c.* incluſive to 960.

But, indeed, the following ſhort Table, by the Aſſiſtance of Addition of Circulates only, will, in every Reſpect, anſwer the ſame Purpoſe.

A TABLE of the Decimal Parts of a L. Sterling.

Qrs.	D. P.	D.	D. P.	S.	D. P.
1	,0010416	9	,0375	9	,45
2	,0020833	10	,0416666	10	,5
3	,003125	11	,0458333	11	,55
D.	D. P.	S.	D. P.	12	,6
1	,0041666	1	,05	13	,65
2	,0083333	2	,1	14	,7
3	,0125	3	,15	15	’75
4	,0166666	4	,2	16	,8
5	,0208333	5	,25	17	,85
6	,025	6	,3	18	,9
7	,0291666	7	,35	19	,95
8	,0333333	8	,4		

Much after the like Method with this, every Practitioner might readily make a Table of the Decimal Parts of any Integer he is inclined to.

Use of the preceding Table.

Find the Decimal Expression equal to 10 S. : 9 D.

	L.	Their Sum	S. D.
1st, 10 *S.* =	,5	= ,5375 =	10 : 9
2dly, 9 *D.* =	,0375		

Find the Decimal Expression equal to 17 S. : 3 D : 3 Qrs.

	L.	Their Sum	S. D. Qrs.
1st, 17 *S.* =	,85	= ,865625 =	17 : 3 : 3
2dly, 3 *D.* =	,0125		
3dly, 3 *Qrs.* =	,003125		

The Reverſe of the laſt is,

To find the Value of any Decimal Fraction in the known Part or Parts of that Integer, to which it refers, whether it be to Money, Weights, Time or Meaſures.

RULE.

Multiply the given Expreſſion by the Number of Units contain'd in the next lower Denomination of that Species to which the given Expreſſion refers; and ſo proceed to multiply the ſeveral Fractional Parts only, by its next lower Denominations, until you come to its loweſt known Parts, and the ſeveral Products ſhall be the ſeveral Parts ſought. The following Example will make the Rule eaſily underſtood.

Example.

Reduce ,865625 to the known Parts of a *L.* Sterling.

20

S. 17,312500 — *Anſw.* 17 S. : 3 D. : 3 Qrs.

12

D. 3,7500 = ,3125 × 12 *D:*

4

Qrs. 3,00 = ,75 × 4 *Qrs.*

See more Examples in the Body of the Book.

FINIS.

A New, Compleat, and Universal SYSTEM or BODY of D imal ARITHMETIC;

CONTAINING,

I. The whole Doctrine of Decimal Numbers, not only the Plain and Terminate, but also such as Repeat or Circulate *ad Infinitum*; and a Plain but perfect Management of bo.., laid down and explained in all the Fundamental Rules of Plain Arithmetic, and by Logarithms.

II. The Application and Use of Decimal Arithmetic in all the Parts or Branches of Arithmetical Science; *viz.* Vulgar Arithmetic, Vulgar Fractions, Duodecimal, and Sexagesimal Arithmetic; also in Algebra and Logarithms, in all which its Excellency and absolute Necessity is fully evinced.

III. The Application and Use in all such Parts of the Mathematics as absolutely require its Assistance; *viz.* Plain Trigonometry, and the Arts depending thereon; as, Navigation, Fortification, Altimetry, and Longimetry; also the Mensuration of all Kind of Superficies and Solid Bodies; and the Arts resulting therefrom; as, Gauging, Surveying, &c.

IV. A New and Compleat Sett of Decimal Tables never before published, shewing by Inspection the Value of all Kinds of Decimals (without the tedious Methods of Reductions hitherto used) to four or six Places of Figures; also all the Common Tables very much inlarged, corrected, and improved; wherein all the Circulating Numbers are marked. With all other Tables of Interest, Annuities, Exchange, &c. necessary to render the Work compleat.

V. An exact and accurate Cannon of Logarithms for natural Numbers. And through the Whole, several Things new and useful, not here expressed.

By BENJAMIN MARTIN.

Printed for J. NOON, at the *White-Hart*, near *Mercers* Chapel, in *Cheapside*. MDCCXXXV.

3/6